

Electrohydrodynamic Systems

In this chapter we review briefly the fundamentals of electrohydrodynamics (EHD), the characteristic EHD dimensionless numbers and the techniques to measure conductivity and electric field, as well as the peculiarities imposed by charging of particles in the classical fluid–mechanical methods for measuring velocity and visualizing fluid flows.

We begin with a brief review of the basic equations, followed by an examination of the physical mechanisms that govern fluid flow through the relevant dimensionless numbers related to electric forces. However, the main emphasis is put on the description of the experimental methods, used to measure the fundamental EHD magnitudes. First, we discuss the basic mechanisms of conductivity, how to measure it, and how to obtain reproducible I – V characteristics. This section also includes a discussion of the techniques to control ion injection. This is followed by a section dedicated to the measurement of mobility. Then we describe the Kerr effect, and how it can be used to measure the electric field in liquids. The last section is dedicated to a description of the difficulties we encounter in the classical techniques of laser Doppler anemometry, and visualization techniques in EHD flows, and how they may be overcome, at least partially. We hope that this chapter will be useful, not only to EHD researchers, but also to practising fluid dynamicists, and to chemical and electrical

21.1 Equations	1318
21.1.1 Electrical Equations	1318
21.1.2 Mechanical Equations	1318
21.1.3 Temperature Equation	1319
21.2 Fluid Statics and Dynamics in EHD	1320
21.2.1 Charge Decay	1320
21.2.2 EHD Statics and Dynamics	1321
21.3 Experimental Methods in EHD	1322
21.4 Conductivity	1323
21.4.1 Conduction Mechanisms in Liquids	1323
21.4.2 Ohmic Versus Non-Ohmic Regime: Practical Estimation of the Transition Voltage	1324
21.4.3 Unipolar Injection	1325
21.4.4 Determination of the Conductivity	1325
21.5 Mobility	1327
21.6 Electric Field Measurement: Kerr Effect	1328
21.7 Velocity	1329
21.7.1 Laser-Doppler Anemometry	1329
21.8 Visualization	1330
21.8.1 Shadowgraph and Schlieren	1330
21.8.2 Tracer Particles	1331
References	1331

engineers who need to understand and apply the principles and experimental techniques of EHD in their work.

Electrohydrodynamic (EHD) systems are hydrodynamical systems subjected to electric fields. Electrical currents are weak, thus, in contrast to magnetohydrodynamics, the liquids are either good insulators (oils, alcohols, purified water), or weak electrolytes (aqueous solutions of conductivities comprised between 10^{-5}

and 1 S/m). Maxwell equations are reduced to the quasi-electrostatic equations. The electrical force term plays a crucial role in Navier–Stokes equation. Also temperature, through the variations induced in mobility, conductivity and permittivity and eventually through Joule heating, plays an important role in electrothermal flows.

21.1 Equations

We present the electrical, mechanical and thermal equations that govern the dynamics of EHD of liquids (for details see [21.1]). We consider a system of characteristic size l and characteristic fluid velocity u_0 .

21.1.1 Electrical Equations

Most systems of interest have linear constitutive relations for the displacement vector $\mathbf{D} = \epsilon \mathbf{E}$, and the field, $\mathbf{H} = \mathbf{B}/\mu$, where \mathbf{E} , \mathbf{B} are the electric and magnetic field, and ϵ and μ are the permittivity and permeability, respectively. A condition that must be met by an electromagnetic system in order to be quasi-electrostatic is that the magnetic field due to the total current density in the system, regardless of its origin, i. e., injection, dissociation, current due to particles, or displacement current, must satisfy the condition $cB_0 < E_0$, where B_0 and E_0 are the characteristic values of the magnetic and electric field, respectively, and c is the velocity of light in the system. This is equivalent to stating that the electric energy density dominates over the magnetic energy density, $(1/2)\epsilon E^2 \gg (1/2)B^2/\mu$.

The scale for B_0 comes from the fourth Maxwell's equation,

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (21.1)$$

and it can originate from the conduction, the displacement current or both. In the first case it is $B_0 \sim \mu l J_0$, and we must have

$$\frac{cB_0}{E_0} \sim \frac{cl\mu J_0}{E_0}, \quad (21.2)$$

and consequently,

$$J_0 \ll E_0 \sqrt{\frac{\epsilon}{\mu}} \frac{1}{l}. \quad (21.3)$$

For typical systems in the laboratory $l \approx 10^{-2}$ m, $\mu \approx \mu_0 \approx 10^{-6}$ H/m and $\epsilon \approx 10^{-11}$ F/m, and we must have $J_0 \ll 3E_0$ in SI units. In insulating liquids this condition is amply satisfied in practical situations as J_0 is in the range of microamperes per square centimeter, and E_0 is several kV per centimeter.

If the displacement current dominates, $B_0 \sim (l/c^2)\omega E_0$, with ω the frequency of the electric field. Then we have

$$\frac{cB_0}{E_0} \sim \frac{l\omega}{c}, \quad (21.4)$$

and we must satisfy $l\omega/c \ll 1$, which is obviously true (for typical systems in the laboratory) for frequencies below microwave frequencies.

Notice that the ratio of the displacement to the conduction current is given by $\epsilon\omega E_0/J_0$. Therefore for ω less (larger) than $\epsilon J_0/E_0$ the conduction (displacement) current dominates. For the particular case of an ohmic liquid, $\mathbf{J} = \sigma \mathbf{E}$, this transition value for ω is σ/ϵ .

The condition $cB_0 \ll E_0$ automatically implies that the electric field is irrotational. In effect,

$$\frac{\partial \mathbf{B}/\partial t}{\nabla \times \mathbf{E}} \ll \frac{\omega l}{c} \ll 1. \quad (21.5)$$

Therefore, in electrohydrodynamics, Maxwell equations reduce to

$$\nabla \cdot \mathbf{D} = q, \quad \nabla \times \mathbf{E} = 0, \quad \frac{\partial q}{\partial t} + \nabla \cdot \mathbf{J} = 0, \quad (21.6)$$

where q denotes the volume charge density.

We must complement these equations with the jump conditions at interfaces, i. e., the continuity of the tangential component of the electric field, and the conservation equation for surface charge density, which is equal to the jump of the normal component of the displacement, $q_s = [\epsilon E_n]$,

$$\begin{aligned} \frac{\partial q_s}{\partial t} + u_n \mathbf{n} \cdot \nabla q_s + (\nabla \cdot \mathbf{n}) u_n q_s \\ + \nabla_s \cdot \mathbf{K}_s + \mathbf{n} \cdot [\mathbf{J}] - u_n [q] \\ = 0, \end{aligned} \quad (21.7)$$

where \mathbf{n} points from phase 1 to phase 2, $\nabla_s = \nabla - \mathbf{n}(\mathbf{n} \cdot \nabla)$, \mathbf{K}_s is the surface current density, and $[f] = f_2 - f_1$ is the the jump of f across the surface (for an alternative derivation to the one given in [21.1] see Appendix A in [21.2]).

21.1.2 Mechanical Equations

For fluid velocities much smaller than the velocity of sound, the fluid may be considered incompressible. Then, the mechanical equations reduce to

$$\nabla \cdot \mathbf{u} = 0, \quad (21.8)$$

for mass conservation, and to the Navier–Stokes equation for momentum conservation

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p_0 + \eta \nabla^2 \mathbf{u} + \mathbf{f}_e + \rho \mathbf{g}, \quad (21.9)$$

where ρ is the mass density, \mathbf{u} the fluid velocity, p_0 the thermodynamic pressure in the absence of the electric

field, η the fluid viscosity, f_e the electrical force density, and the last term is the gravitational force per unit volume with g the gravity acceleration.

The electrical force density is $f_e = \nabla \cdot \mathbf{T}^e$, with \mathbf{T}^e given by

$$\mathbf{T}^e = \left\{ -\mathbf{E} \cdot \mathbf{D} + \int_0^D \left[\mathbf{E} - \rho \left(\frac{\partial \mathbf{E}}{\partial \rho} \right)_{T,D} \right] dD \right\} + D\mathbf{E}. \quad (21.10)$$

For an electrically linear medium the components of \mathbf{T}^e are

$$T_{ij}^e = \epsilon E_i E_j - \frac{1}{2}(1-a)E_k E_k \delta_{ij} \quad (21.11)$$

where $a = (\rho/\epsilon)(\partial\epsilon/\partial\rho)_T$ is the electrostriction parameter. In this case we recover the classical expression for the electric force density

$$f_e = \nabla \cdot \mathbf{T}^e = q\mathbf{E} - \frac{1}{2}E^2\nabla\epsilon + \nabla \left(\frac{1}{2}\epsilon a E^2 \right). \quad (21.12)$$

The term $q\mathbf{E}$, called the Coulomb force, is the force per unit volume on a medium containing free electric charge. This is the strongest EHD force term and usually dominates when direct-current (DC) electric fields are present. The next term, called the dielectric force, is due to the force exerted on a nonhomogeneous dielectric liquid by an electric field. This is usually weaker than the Coulomb force and only dominates when an alternating-current (AC) electric field with a period much shorter than the charge relaxation time and/or the ionic transit time, is applied. The last term, called the electrostrictive force is the gradient of a scalar and is treated as a modification to the fluid pressure.

21.1.3 Temperature Equation

The derivation of the temperature T equations is lengthy [21.1, 3]. The result is

$$\begin{aligned} & \rho \left\{ c_V - T \frac{D^2}{2\rho} \left[\frac{\partial^2}{\partial T^2} \left(\frac{1}{\epsilon} \right) \right]_{\rho} \right\} \frac{dT}{dt} \\ &= \frac{T}{\rho} \left(\frac{\partial p}{\partial T} \right)_{\rho,D} \frac{d\rho}{dt} - \nabla \cdot \mathbf{J}'_0 + \Phi \\ &+ \mathbf{E} \cdot \mathbf{J}' + T \left[\frac{\partial}{\partial T} \left(\frac{1}{\epsilon} \right) \right]_{\rho} D \cdot \frac{D\mathbf{D}}{dt}, \end{aligned} \quad (21.13)$$

where c_V is the specific heat at constant volume, \mathbf{J}'_0 , \mathbf{J}' are the heat flux and current density measured in

the rest frame of the fluid element, respectively, and $\Phi = 2\eta e_{ij}^2$ with $e_{ij} = (1/2)[(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)]$ is the heat generation due to viscous dissipation. If we neglect thermoelectric effects, the heat flux is given by $\mathbf{J}'_0 = -\tilde{k}\nabla T$, with \tilde{k} the thermal conductivity in the presence of the electric field. The dependence of the thermal conductivity on the electric field has not been reported in the literature, and is probably negligible, therefore we take $\tilde{k} = k$, where k is the usual thermal conductivity. The equation for temperature may be also written in terms of the pressure p instead of the mass density ρ as the independent variable for the entropy, since most processes in liquids take place at constant pressure [21.3].

For moderate heating the equation of state for the mass density may be approximated by $\rho = \rho_0[1 - \alpha(T - T_0)]$, with ρ_0 being the reference density at temperature T_0 and α the volume expansion coefficient evaluated at the reference temperature. The permittivity ϵ is also assumed to vary linearly with temperature, $\epsilon = \epsilon_0[1 - e_1(T - T_0)]$, with e_1 being the relative derivative of the permittivity with respect to temperature. At ambient temperature $e_1 \approx 10^{-3}$. For frequencies below the microwave region, which are typical in EHD, the molecular polarization is in equilibrium with the field, and dielectric heating is not present. Since under the Boussinesq approximation the fluid properties η , k , c_V and α are assumed to be constants, and the contribution from viscous dissipation is neglected, the temperature equation for processes at constant pressure reduces to

$$\rho_0 c_p \frac{dT}{dt} = k \nabla^2 T + \mathbf{E} \cdot \mathbf{J}', \quad (21.14)$$

where c_p is the specific heat at constant pressure.

For highly insulating liquids, the currents are of the order of μA or less, and the contribution of Joule heating to the energy equation may be neglected. The energy equation is decoupled from the electrical and mechanical equations. Therefore, the solution depends only upon the boundary and initial conditions imposed on the temperature T . We have

$$\rho c_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = k \nabla^2 T. \quad (21.15)$$

However, the temperature cannot be ignored as its distribution affects the dynamic behavior of the system through the buoyancy and electric forces. In addition the temperature also affects the charge density distribution through the variations induced in the permittivity, the conductivity and the mobility. Reference [21.4] has a discussion of these effects in some detail.

21.2 Fluid Statics and Dynamics in EHD

The importance of the principal electrical and mechanical effects depends on the ratio of charge decay and electrical and mechanical time constants [21.5], or equivalently on the pressure or forces ratios of mechanical and electrical origin [21.6], or alternatively through the dimensionless EHD equations [21.1]. In this section we discuss the effect that electrical terms have on the statics and dynamics of fluids, but first we need to discuss the evolution of the charge density.

21.2.1 Charge Decay

The evolution of the charge density is given by

$$\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{J} = 0. \quad (21.16)$$

Ohmic Regime

For liquids in the ohmic regime $\mathbf{J} = \sigma \mathbf{E} + q\mathbf{u}$, where \mathbf{u} is the liquid velocity. Then,

$$\frac{dq}{dt} + \frac{\sigma}{\epsilon} q = 0, \quad (21.17)$$

where $d/dt = \partial/\partial t + \mathbf{u} \cdot \nabla$. The solution is $q(t) = q(0)e^{-t/\tau_e}$. Therefore, in the local rest frame of the fluid, the charge relaxes exponentially with a characteristic time $\tau_e = \epsilon/\sigma$. However, even if the interfaces are initially neutral, charges will accumulate at them because for steady conditions the normal current density has to be continuous, $\sigma_1 E_{n1} = \sigma_2 E_{n2}$, which implies that $q_s = \epsilon_2 E_{n2} - \epsilon_{n1} E_{n1}$ will in general be different from zero. The charging time τ is a combination of the relaxation time of each phase and geometrical factors. An example of the importance of this relaxation time in an EHD problem is the electrified bouncing ball (see, for example [21.7]).

In AC electric fields of period $T = 2\pi/\omega$ much smaller than the relaxation time the charge density at interfaces cannot change, and consequently the average of the Coulomb electric surface forces is equal to zero. Therefore, when we want to avoid the effect of charges the common practice is to use fields of frequency much larger than the charge relaxation time of any of the phases present in the system. For a practical example see [21.8].

Unipolar Injection Regime

Metallic point electrodes, sharp edges, and fine wires submerged in insulating fluids have intense local electric

fields at their surfaces because of the enhancing effects of curvature. This originates electron injection, either from the metal to molecules in the fluid or viceversa. As a result, ions are created and move under the influence of the electric field. For the sake of simplicity, consider that only one ionic species, taken to be positive, is injected into a perfectly insulating liquid $\sigma = 0$. The constitutive law for the current density is

$$\mathbf{J} = q\mathbf{KE} - D\nabla q + q\mathbf{u}, \quad (21.18)$$

where \mathbf{KE} is the velocity of the ions relative to the fluid, called the drift velocity, and K is the ion mobility.

Let us compare the diffusion term to the drift term. It is

$$\frac{D\nabla q}{q\mathbf{KE}} \sim \frac{D}{KV} = \frac{0.025}{V}, \quad (21.19)$$

where we have used Einstein's relation $D/K = k_B T/e$ (for an ambient temperature of $T \approx 300$ K). Consequently, diffusion will be negligible compared to drift unless the potential difference is of the order of 0.025 V. Usually this will only happen in thin layers, either boundary layers or internal layers, where diffusion is required to match the outer or mainstream solutions of the diffusion-free problem. For example, it will be important in the dynamics of the charged double electrical layers in electrolytes. However, for highly insulating liquids diffusion may be safely neglected. Then, the charge conservation equation can be written as

$$\frac{\partial q}{\partial t} + (\mathbf{u} + K\mathbf{E}) \cdot \nabla q + \frac{K}{\epsilon} q^2 = 0, \quad (21.20)$$

or equivalently $d/dt(1/q) = K/\epsilon$, when we move with the ion velocity $\mathbf{KE} + \mathbf{u}$. This equation has the exact solution $q = q_0/(1 + t/\tau_C)$ with $\tau_C = \epsilon/Kq_0$ being the algebraic bulk relaxation time. This solution is valid on $d\mathbf{r}/dt = \mathbf{KE} + \mathbf{u}$, so that unless a given element of liquid can be traced via a particle line to a source of charge, it will support no bulk charge density. The algebraic charge relaxation time τ_C depends on the initial level of charge, and after several decay times, the charge density reaches a limiting value ϵ/Kt independent of the initial conditions. It is important to note that charge relaxation due to Coulomb repulsion is much slower than the classical exponential relaxation time. This explains why insulating fluids may accumulate charges easily. A classical example is fuel, to which we must add electrostatic additives that increase their conductivity and facilitate the reduction of their charges in order to avoid electrostatic hazards.

The ratio between Coulomb repulsion and drift, $(K/\epsilon)q^2/KE \cdot \nabla q$, is given by the dimensionless number $C = \tau_i/\tau_C = q_0 l^2/\epsilon V$ where $\tau_i = l^2/KV$ is the transit time, i. e., the time for an ion to cross the typical length l . For low (large) values of C the characteristic time that enters into the problem is the transit time τ_i (Coulomb repulsion time τ_C). Small values of C indicate that the electric field is determined by the external power supply connected to the electrodes in the fluid. On the contrary, for high values of C the electric field is strongly dependent on the charges that are either inside or on the surface of the fluids.

The quotient u/KE is the ratio of the transit time to the mechanical convection time. This ratio is called the electric Reynolds number Re , in analogy to the Reynolds number, which is the ratio of viscous time to inertial time. For high values of this number the charge may be considered to be attached to the liquid, i. e., the liquid behaves like a good insulator. On the contrary, for small values of Re , the liquid may be considered to be a conductor.

Mixed Model

It is quite common to consider the mobility model with a background liquid conductivity

$$\mathbf{J} = qK\mathbf{E} + \sigma\mathbf{E} + q\mathbf{u}. \quad (21.21)$$

The ratio of conduction current to mobility current is $\sigma E/KqE \sim \sigma/Kq_0 = \tau_C/\tau_e = C_0$. Large values of C_0 indicate that the liquid is basically a conductor, the opposite being true for $C_0 \ll 1$. Another way to write this important number is to take into account that $q_0 \sim \epsilon V/l$ (from Gauss's law). Then $C_0 = \sigma l^2/K\epsilon V$, which implies that for large systems or low values of the applied voltage the liquid will behave as an ohmic conductor.

21.2.2 EHD Statics and Dynamics

There may exist solutions to the EHD equations that are time independent. The simplest one is the hydrostatic solution $\mathbf{u} = 0$. For this solution to be possible, the electric stresses must be normal to any fluid interface present in the system, and in the bulk it must satisfy

$$\nabla \times \left(q\mathbf{E} - \frac{1}{2}E^2\nabla\epsilon + \rho\mathbf{g} \right) = 0. \quad (21.22)$$

The interfacial conditions are always met for perfectly insulating liquids, since for them it is $q_s = 0$, and for perfect liquid conductors because the electric field is always normal to the interface. As previously indicated,

for semi-insulating liquids there is a necessity to use AC fields of high enough frequency if we want to impose zero Coulomb electric force density at the interfaces. In general, these conditions, plus the condition in the bulk, are met only in highly symmetrical geometries (planar, cylindrical or spherical layers). Let us then consider the general case of dynamics.

Interfacial Dynamics

In the classical boundary conditions we have to add, to the jump across the interface of the mechanical stress tensor, the jump of electrical stresses $[\mathbf{T}_e \cdot \mathbf{n}]$. The order of magnitude of these forces acting at the interface, is γ/R for capillary forces, where γ is the surface tension and R the minimum radius of curvature; $[\epsilon E_n E_t] \sim [\epsilon E_0^2]$ for the electrical terms; $[\rho]gl$ for the gravitational force, where l is the length scale for the interface along the gravity acceleration. Therefore, the system behavior is determined by the following two dimensionless numbers: the Bond number $Bo \equiv [\rho]gL R/\gamma$, defined by the ratio between the gravitational and the capillary forces; the electric bond number $B_e \equiv [\epsilon E_0^2]R/\gamma$, which is the analog using the electrical force instead of the gravitational force; and the number $W_e \equiv [\epsilon E_0^2]/[\rho]gL$, defined by the ratio of the electrical force to the gravitational one. This parameter governs the equilibria when capillary effects are small compared to the electrical and gravitational forces. When only the normal stresses are different from zero and $Re \gg 1$, the fluid may be considered as ideal, and the electric Bond number and W_e or a combination of both will govern the dynamics of the system. Well-known examples are linear and nonlinear waves and instabilities in horizontal interfaces and jets [21.2, 9]. Viscosity must always be considered when the liquids are neither perfectly conducting nor perfect insulators, since it is the only mechanism that balances the electrical stresses at the interface. The classical Ohnesorge number $Oh \equiv [\rho\nu]u_0 R/\gamma h_0$, which compares the viscous terms with the capillary terms, enters into the problem.

An example of a steady-state solution is *Quincke's rotor*. This is a solid, highly insulating, cylinder immersed in a viscous, conducting, liquid. An electric field, uniform and of value E_0 at large distances of the cylinder, is imposed perpendicular to its axis. Due to the symmetry of the system, there is an equilibrium state, but this state is unstable. A slight rotation produces an asymmetrical distribution of charge. In this case, there is a net torque on the cylinder and it begins to rotate. Assuming that the system is viscous dominated, the angular

velocity may be estimated as

$$\rho v \frac{u_0}{R} \sim \rho v \Omega \sim q_s E_s. \quad (21.23)$$

The tangential electric field is, in this system, of the same order as the external field, whereas the surface charge density can be obtained from the surface charge conservation equation. For steady conditions the latter can be written as $\Omega(\partial q_s / \partial \theta) + [\sigma E_r] = 0$, and from here, the order of magnitude for q_s is $q_s \sim (\sigma E_0) / \Omega$. Finally, we obtain

$$\Omega^2 \sim \frac{\sigma E_0^2}{\rho v} = \frac{1}{t_{ev} \tau_e} \quad (21.24)$$

where $t_{ev} = \rho v / \epsilon E_0^2$ is the electroviscous relaxation time (see below), and $\tau_e = \sigma / \epsilon$ is the charge decay time. For corn oil ($\epsilon \approx 3.1 \times 10^{-11}$, $\sigma \approx 5 \times 10^{-11}$, $\rho v \approx 5 \times 10^{-2}$ in SI units) and an applied electric field around 1 kV/cm, the resulting rotation period is of the order of one second. This phenomenon has been observed in practice, and is responsible for the rotation of small particles immersed in insulating liquids and subjected to DC fields [21.10–12].

Bulk Dynamics

The dynamics is governed by Navier–Stokes equation. In addition to the classical nondimensional numbers in fluid mechanics, we have to consider new electromechanical parameters defined as the ratios of the Coulomb force, or dielectric force, to the pressure, viscous or inertial forces. These numbers may be written in a variety of ways depending on the physical mechanisms that determine the velocity field, the charge density and the gradient of permittivity. Since $q_0 \sim \epsilon E / l$, the ratio of the Coulomb term to the dielectric term of the electric force is given by $\epsilon / (l \nabla \epsilon)$. In general, the Coulomb force is typically much larger than the dielectric force; for example, when the variation of the permittivity is induced by a temperature gradient it is $\nabla \epsilon = 10^{-3} \Delta T \epsilon$, and therefore this ratio is equal to $10^3 / l \Delta T$. For temperature differences, which are typically of a few degrees Celsius, and for laboratory systems this ratio is much greater than one. In general, only for those situations for

which q_0 is negligible, and $\nabla \epsilon$ different from zero, is the dielectric force the dominant electric force.

Consider the transient motion of a liquid driven solely by the electrical forces. In the viscous regime $\eta \nabla^2 \mathbf{u} \sim \mathbf{f}_e$. Equating both terms, we find a natural time scale $\tau_{ev} = \eta / \epsilon E^2$, called the electroviscous scale, which plays a significant role in these transient viscous regimes. This same ratio tells us that for steady conditions the typical scale for the velocity is $u_0 = \epsilon V^2 / (\eta l)$.

In inertial flows, the most frequently encountered in nature or in industrial processes, the acceleration term is comparable to the electric force. In other terms, the electrical energy is converted to kinetic energy of the liquid. This implies that $(1/2) \epsilon E^2 \simeq (1/2) \rho u_0^2$, and consequently that the typical liquid velocity is given by $u_0 \simeq (\epsilon / \rho)^{1/2} E_0 = K_h E_0$. The liquid velocity is proportional to the electric field, and the constant $K_h = (\epsilon / \rho)^{1/2}$ is called the hydrodynamic mobility. It is normal to define a dimensionless number $M = (\epsilon / \rho)^{1/2} / K$ as the ratio of two mobilities, the hydrodynamic mobility, and K , the true ionic mobility. Taking into account *Walden's rule*, i. e., $K \eta \sim \text{const.}$, we see that M is proportional to viscosity. Since $u \simeq M K E$ and for liquids M may vary from 4 to 400, ions are entrained by the motion of the liquid, contrary to the case of gases where M is less than one. For charged liquids, the time of flight of ions may be much larger than the algebraic charge relaxation time or the ionic transit time, and the final charge distribution may be strongly affected by the fluid motion. As a consequence, the electric field is modified, which in turn changes the fluid velocity, thus making these types of problems particularly difficult. For further discussion of the effect of electric forces on the dynamics of the system see the references cited at the beginning of this section.

Electrothermal flows are those flows due to the variations in the mobility K , conductivity σ , and permittivity ϵ due to a thermal gradient, either externally imposed or originating through Joule heating. These variations give rise to variations in the charge density, and therefore in the electrical force, thus affecting fluid flow. For the underlying physics of these subtle flows we refer the reader to [21.4] for the case of insulating liquids, and to [21.13] for the case of weak electrolytes in microfluidics.

21.3 Experimental Methods in EHD

The experimental study of EHD systems is besieged by several fundamental difficulties. One is the extreme dif-

ficulty of determining the nature of ions and to predict their density distributions due to chemical complex-

ity. The result is that the constitutive equations for the current density are rarely known. Therefore, we must work with simplified models. Well-known examples of such models are the conduction and the mobility models previously discussed. Another important problem is the difficulty of measuring charge density, which is highly desirable as electrical forces depend on it. A third difficulty is avoiding or minimizing charging effects on particles used as tracers, as well as electrothermal flows when we use Schlieren techniques.

The determination of the nature of ions has been partially solved in gases using spectrography. In liquids the unavoidable presence of minute traces of impurities that

have a huge effect on conductivity makes the problem untractable, unless

1. we dissolve a known salt or a compound, such that the dominant ions are known,
2. we inject massively one type of ion, or
3. we keep the liquid recirculating through a electro-dialysis cell.

Let us discuss then how we model conduction, injection and the current density and the problems encountered in its measurement. Then, we will discuss the determination of the electric field, and finally the measurement of the velocity field and its visualization.

21.4 Conductivity

The experimental determination of the conductivity of low-conductivity liquids is one of the most challenging issues that researchers in electrohydrodynamics have to confront. The purpose of this section is to describe the difficulties often encountered in measuring the conductivity and how they can be overcome, or, at least, their effects reduced.

21.4.1 Conduction Mechanisms in Liquids

Low-conductivity liquids, such as hydrocarbons, do not self-ionize, and the charge carriers are ions originating from the spontaneous dissociation of tiny amounts of ionizable substances present in the liquid. In the case of practical grade liquids the ionizable substances spring from the unavoidable impurities. It is, however, a common practice to add some ionophores in order to increase and stabilize the conductivity of these liquids.

For a weak electrolyte AB dissolved in a liquid the following chemical equilibrium determines the number of dissociated ions:



There is a constant associated with the forward direction, the dissociation constant K_D , whereas for the backward reaction the constant is the recombination constant K_R . The dissociation constant can be expressed as (see [21.1] and references therein)

$$K_D = 3e \frac{(K_+ + K_-)}{4\pi\epsilon a^3} e^{-U/k_B T} \quad (21.26)$$

and the recombination constant as

$$K_R = e \frac{(K_+ + K_-)}{\epsilon} \frac{1}{1 - e^{-U/k_B T}} \quad (21.27)$$

where $U = e^2/(4\pi\epsilon a)$, a is the minimum distance of approach between the centers of the ions, and K_+ and K_- are the mobilities of the cations and anions, respectively.

The number of ions in equilibrium is obtained from the balance

$$K_R n_+ n_- = K_D c \quad (21.28)$$

where c is the concentration of salt, n_+ is the concentration of positive ions and n_- is the concentration of negative ions. Hence

$$n_+ = n_- = n_0 = \sqrt{\frac{K_D c}{K_R}} = A e^{-e^2/(4\pi\epsilon a k_B T)} . \quad (21.29)$$

This number is very sensitive to the value of the dielectric constant ϵ , which appears in the argument of the exponential. This fact explains the dependence of the conductivity on the dielectric constant.

If the applied electric field is not very high, this equilibrium is not perturbed and determines the number of charge carriers. Also, the liquid volume remains electrically neutral and the electric field acting on the charge carrier is the applied field. Therefore the regime of conduction is ohmic and the current density becomes

$$j = e(n_+ K_+ + n_- K_-) E = \sigma E \quad (21.30)$$

where σ is the conductivity

$$\sigma = e(K_+ + K_-)n_0. \quad (21.31)$$

Dependence of the Conductivity on the Dielectric Constant

The effect of the dielectric constant on the equilibrium (21.25) is determined by its role on the exponential of (21.29). For low dielectric constants the exponential is small and so is the density of charge carriers. In this way the density of charge carriers is an increasing, and very sensitive, function of the dielectric constant. For example, a change in ϵ from 2 to 20 results in an increase of 10 orders of magnitude in n_0 . The physical origin of this effect is the screening of the electric field of an ion by the molecular dipoles of the liquid. This screening is almost absent in low-polar liquids, and the ion pairs are tightly bound in that case.

Equation (21.29) allows one to obtain an empirical law for the dependence of the conductivity on the dielectric constant [21.14]:

$$\sigma \propto n_0 \propto e^{-e^2/(4\pi\epsilon\epsilon_0 k_B T)} \rightarrow \log \sigma = A - \frac{B}{\epsilon}, \quad (21.32)$$

where A and B are constants. This correlation fits a large number of experimental data reasonably well (Fig. 21.1).

Dependence of the Conductivity on the Concentration of Solute

For a weak electrolyte in a low-polar liquid, (21.29) and (21.31) yield a dependence of σ on c of the type

$$\sigma \propto \sqrt{c}. \quad (21.33)$$

And this is the case for a great number of salts when added to a low-polar liquid such as tri-isoamyl-ammonium picrate (TIAP) or tetra-butyl-ammonium chloride (TBA) in hydrocarbons.

However, the addition of surfactants (for example, AOT, di-2-ethylhexyl sulfosuccinate) may give place to another kind of dependence [21.15]. In effect, surfactants are compounds whose molecules have a long non-polar tail (usually an alkyl radical) and a polar head. When surfactants are dissolved in a non-polar medium they tend to form inverse micelles. These are groups of molecules, formed from 10–20 molecules, with their polar heads oriented towards the center of the group and the non-polar tails pointing towards the solvent volume. The environment in the core of the micelle is a polar one, and it is a convenient place for an ion that otherwise could not be stable in the volume of the solvent.

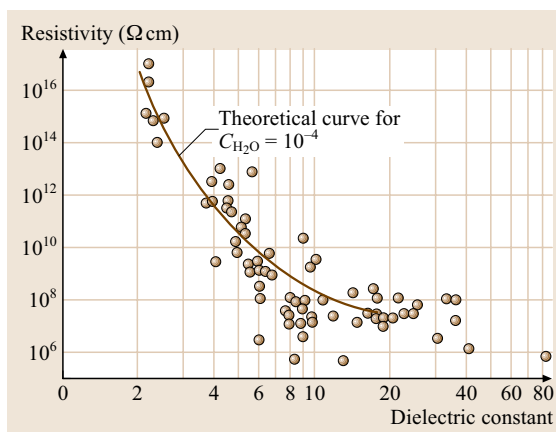


Fig. 21.1 Resistivity as a function of dielectric constant for different liquids (after [21.14])

The formation of micelles, which takes place above a certain concentration referred to as the critical micellar concentration, helps to increase the conductivity. However, in this case the kinetics of charge carrier formation is different to the case of bare ions (21.25). The equilibrium is replaced by



because the anion and the cation reside in the interior of their respective micelles. If c is the concentration of the surfactant, the conductivity is in this case proportional to c :

$$\sigma \propto c, \quad (21.35)$$

which is the same behavior as that expected for a strong electrolyte in an aqueous media.

21.4.2 Ohmic Versus Non-Ohmic Regime: Practical Estimation of the Transition Voltage

The ohmic regime, in which the electric current density is proportional to the applied electric field, is established only if the electric field scarcely perturbs the thermodynamic equilibrium between the charge carriers and the neutral molecules that originate them. Associated to this thermodynamic equilibrium there is a characteristic time τ_t , which is the time needed to recover the equilibrium after the perturbation. This thermodynamic time is one half the electric relaxation time of the liquid $\tau_e = \epsilon/\sigma$ [21.1]. On the other hand the transit time of a charge carrier between two electrodes a distance d apart is $\tau_i = l^2/(KV)$. This is also the time a local per-

turbation of the equilibrium needs to propagate across the gap between the electrodes.

The voltage above which the ohmic regime is no longer expected is estimated putting $C_0 = 1$, i. e., equating the time of transit and the electric relaxation time:

$$\frac{\epsilon}{\sigma} = \frac{l^2}{KV}. \quad (21.36)$$

This gives

$$V_s = \frac{\sigma l^2}{K\epsilon} \quad (21.37)$$

for the transition voltage, known as the saturation voltage.

Well above the saturation voltage, the ions are swept from the volume at a rate that prevents their recombination. The current is then limited by the rate at which the ions are created in the liquid volume and becomes independent of the applied voltage. This current is the saturation current and its value is

$$j_s = eK_D c d. \quad (21.38)$$

However, this is not the end of the story and at even higher voltages the field-enhanced dissociation [21.16] gives place to an increase in the current that is a nonlinear function of the applied voltage.

The notion of saturation voltages deserves two commentaries. First, the transition between ohmic and non-ohmic regime is a continuous one, from a clear linear relation between the electric current and the applied voltage to a nonlinear relation that depends on the kind of process that dominates the electric conduction above saturation. There is, in general, no discontinuity in the experimental current–voltage characteristics and no unambiguous way of assigning a definite voltage to the transition.

Secondly, if the voltage is applied during very short periods of time, compared to the dielectric relaxation time, the instantaneously measured current should correspond to the stationary ohmic value. This is due to the fact that the equilibrium has not yet been appreciable disturbed in that case.

21.4.3 Unipolar Injection

One of the most important mechanisms of charge transport above the saturation voltage is charge injection. This is also one of the most common process for originating space charge in the volume of insulating liquids. Under non-controlled circumstances charge injection is

erratic, non-steady and unipolar or bipolar. There are several techniques to control injection.

The electron injection technique [21.17] uses an electron beam in a vacuum chamber. The electron beam sweeps the liquid surface in both directions analogously to a television set. The voltage of a grid above the liquid surface enables one to measure the surface voltage. This technique has been successfully applied to the study of EHD instabilities. However it only works on a free surface and in a vacuum.

Corona discharge from a needle or a blade can be used to induce injection over a free liquid surface. The injection in the liquid is strong, whether the corona current is space charge limited or not, because the mobility of ions in liquids is much smaller than the mobility of ions in air. The major drawbacks of this technique are that there is no direct way to measure the voltage on the liquid surface and that the injection is not uniform. The major advantage is that it is very easy to implement [21.18].

Ion exchange membranes are widely used in electrodialysis. They are also of interest for injection experiments. Covering the electrodes with ion exchange membranes creates a reservoir of ions suitable for injection when the electric field is applied. In this way a strong injection is usually obtained [21.19, 20].

The use of additives increases the conductivity of the liquid, but it may help to induce and stabilize injection. For example, the salt tri-isoamyl-ammonium picrate (TIAP) has often been used as such an additive. The level of injection achieved with this technique is low to moderate [21.21]. The addition of iodine with electrodes made from titanium has also turned out to be a good technique to obtain stable and reproducible injection [21.22].

Finally sharp electrodes immersed in the liquid may cause stable injection at high voltages without the use of any additive. Electrohydrodynamic plumes emanating from the sharp electrode have been observed in this way [21.23].

21.4.4 Determination of the Conductivity

Current–Voltage Characteristics

The most widely used method to measure the conductivity of low-conducting liquids is to obtain a current–voltage characteristic and estimate the conductivity from the linear part of the curve. For very low-conducting liquids the gap between the electrodes has to be small enough in order to have a detectable current for moderate applied voltages. For example a li-

quid of conductivity of the order of 10^{-12} S/m will yield a current of the order of picoamperes when subjected to a 100 V difference across a gap of 1 mm and 25 cm^2 of electrode surface. For this reason a very sensitive electrometer or picoammeter is required.

Concerning the cell design several precautions must be taken into account [21.25]:

- Since the liquid to be tested may have a very low conductivity the electric field is not totally confined to the liquid volume. This is the reason why a guard ring is desirable. This ensures that all the measured current originated in a region of uniform field.
- It is convenient to have a variable gap between the electrodes. One of the problems one usually confronts when measuring the conductivity is to assure that the conduction regime is ohmic; only then it is meaningful to extract a value of conductivity. In the ohmic regime, the current must decrease as the inverse of the electrode gap. This provides an additional criterium, besides the linearity with the applied voltage, to decide whether the ohmic regime is established or not. On the other hand a variable electrode gap facilitates mobility measurements, as we will see below.
- Materials are also very important. Chemical reaction at the electrodes may be the source of additional charge carriers. Although stainless steel is widely used as a material for the electrodes it may be interesting to have a set of electrodes made from different materials. Also the insulating parts are of concern. Although Teflon has often been used, because of its outstanding insulating properties, its porosity makes it unsuitable for parts in contact with the liquid, as it can be a reservoir for impurities.

After several current–voltage curves have been taken, and the correct dependence with voltage and distance have been found, the estimated conductivity should be used to determined from (21.37) the saturation voltage, confirming that one has been working well below this voltage. Figure 21.2 shows a typical current–voltage characteristics where a linear part is distinguishable at low voltages. After the saturation voltage, I increases faster than ohmically, a fingerprint of an injection process.

Commercially Available Apparatuses

There are commercially available apparatuses that work properly up to conductivities of the order of 10^{-8} S/m. However no one goes beyond this value. To the knowl-

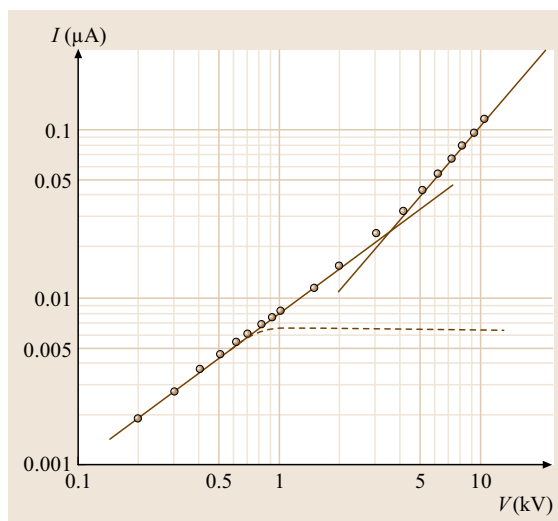


Fig. 21.2 I – V characteristics. From the linear part the conductivity can be estimated. The horizontal line corresponds to the saturation voltage. The steeper slope above the saturation voltage is characteristic of an injection process (after [21.24])

edge of the authors, the only commercially available conductivimeter is the one manufactured by IRLAB based on a method developed in Grenoble by Tobazéon et al. [21.26,27].

This apparatus exploits the fact that the current measured just after the voltage has been established corresponds to the ohmic value, provided the time at which the measure is achieved is small compared to the transit time of ions. The liquid is placed between two concentric cylindrical electrodes and subjected to a square-wave voltage of amplitude 10 V and frequency 0.5 Hz. The conduction current is measured during a short time interval during each period and averaged over a suitable number of periods. In this way their builders claim that the apparatus is able to measure conductivities as low as 10^{-15} S/m.

Some Additional Drawbacks

On top of the already commented difficulties some additional problems are encountered when measuring the conductivity of low-conducting liquids. The two major problems are stability and reproducibility.

By stability we mean the constancy of the measured value in time. Usually this is not the case and the conductivity of the liquid may vary in time by as much as an order of magnitude. The reasons for this behavior are to be found in the sensitivity of the conductivity

to the presence of impurities. Even a small amount of substance can change the conductivity notably.

In addition, the obtained values are not reproducible. This is also related to the chemical composition of the sample. Only a very careful purification of the liquid,

followed by complete control of the additives and their purity can give a reproducible value.

We believe that, in general, for practical insulating-grade liquids it is impossible to give a value of the conductivity with less than 30% error.

21.5 Mobility

The mobility of charge carriers is another of the important physical properties in electrohydrodynamics. Two

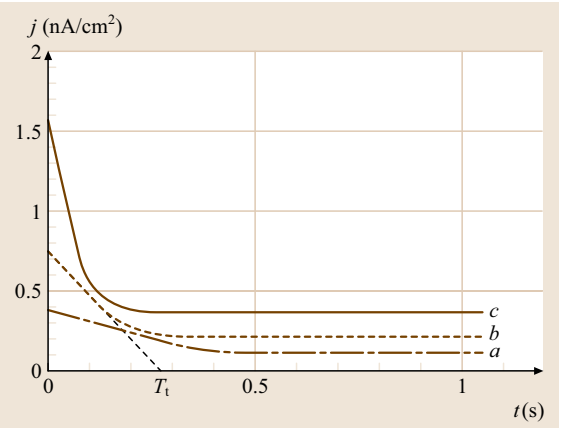


Fig. 21.3 Transient current after a step voltage has been applied. The voltage is greater than the saturation voltage and there is no other source of ions except dissociation in volume (after [21.15])

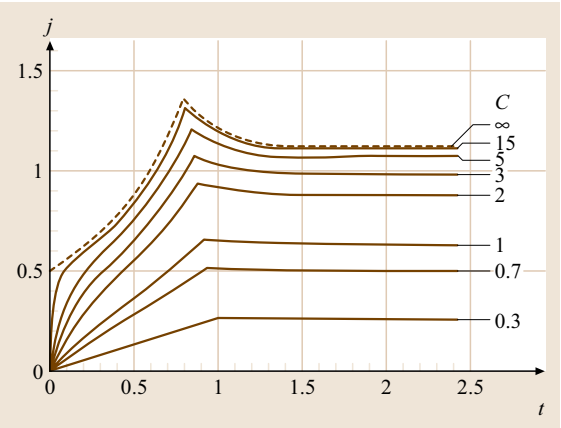


Fig. 21.4 Transient current after a step voltage has been applied when injection is taking place from one of the electrodes (after [21.28])

methods, closely related, are of practical importance in the measurement of mobility.

The first method is the so-called time-of-flight method [21.29, 30] (Chap. 3.7.3). This method is based on the measurement of the time the ions need to cover the distance between two electrodes. In [21.30] the ions are injected from a point and directed towards the collecting electrode through two consecutive grids. Initially the voltage of the different electrodes, the two grids, the collector and the injector, are arranged in such a way that the ions go from the injector to the collector and a constant current is established in the collector circuit. Then the voltage is switched in such a way that no more ions enter the drift space, i. e., the space between the second grid and the collector, whereas the voltage difference between this second grid and the collector is maintained. The time elapsed from switching off the voltage until the extinction of the collector current is the time of flight of the ions from the grid to the collector. From this time the velocity of the ions, and from it the mobility, can be calculated.

The second method makes use of the current transient observed after a step voltage is applied between two parallel electrodes. In principle two types of transient

Table 21.1 The mobility of several ions in different solvents

	$K \text{ (m}^2/\text{Vs)}$
TIAP in cyclohexane	1.3×10^{-8}
Bu ₄ NPi in cyclohexane	1.4×10^{-8}
TIAClO ₄ in cyclohexane	1.2×10^{-8}
DAP in cyclohexane	1.05×10^{-8}
TIAP in hexane	4×10^{-8}
TIAP in benzene	3.3×10^{-8}
AOT in cyclohexane	6×10^{-9}
CrAc in cyclohexane	3×10^{-9}

TIAP: tri-isoamyl-ammonium picrate, Bu₄NPi: tetra-butyl-ammonium picrate, TIAClO₄: tri-isoamyl-ammonium perchlorate, DAP: dodecylamine propionate, AOT: sodium di-2-ethylhexyl sulfosuccinate, CrAc: alkyl-salicylate chromate

are expected if the applied voltage is above saturation. Figure 21.3 is typical of a volume conduction above saturation: the current is a decreasing function of time that reaches a stationary value corresponding to the saturation current. In this case the time of transit τ_i is the time of flight of the ions, $\tau_i = l^2/(KV)$. Figure 21.4 is typical of an injection process [21.28]. In this case it is the time at which the maximum is reached that corresponds to the time of flight of ions. This peak is well defined only for strong injection and cannot be well defined for weak injections, resulting in an ambiguous measurement.

The major drawback of this second method is that often the transit currents are not easy to interpret and do not fall clearly into one of the types mentioned. This can

be due to various reasons. First it is possible that more than one type of carrier dominates the conduction process, producing a blurring of the curves. Secondly other process may be taking part: field-enhanced conduction and bipolar injection, for instances. In order to guarantee that what it is measured is a mobility it is very convenient to make use of a measurement cell with a variable gap. In this way the correct dependence of the time of flight on the voltage and the distance can be tested.

Table 21.1 shows the value of the mobility for various ions in several solvents. This table has been elaborated from different sources, mainly [21.21, 31, 32]. Another table can be found in Chap. 3.7.3 (Table 3.33) of this handbook.

21.6 Electric Field Measurement: Kerr Effect

To our knowledge, charge density in liquids has only been indirectly determined through measurements of the electric field. To measure the latter, we use the fact that many dielectric liquids become birefringent when stressed by a high electric field so that an incident linearly or circularly polarized light propagating through the liquid becomes elliptically polarized. This electro-optic effect, known as the Kerr effect, depends quadratically on the transversal components of the electric field. If we denote by E the applied electric field,

then

$$\Delta n = (n_{\parallel} - n_{\perp})\lambda BE_T^2, \tag{21.39}$$

where n_{\parallel} and n_{\perp} are the refraction indexes along the parallel and perpendicular directions of the electric field, respectively, λ is the free-space wavelength of light, E_T is the component of the electric field in the plane perpendicular to the direction of light, and B is the Kerr constant. Table 21.2 gives the value of the Kerr constant for several liquids.

Suppose that the electric field is constant and we know its direction. Then, with a source of white light, and the help of a polarizer, we may select a linear wave propagating in the direction perpendicular to the electric field, with electric vector e , making an angle α with E . After crossing the cell of length l , the phase shift between the extraordinary (parallel) and the ordinary (transversal) rays is given by $\phi = 2\pi B l E^2$, and the wave is elliptically polarized. Placing an analyzer that only lets linearly polarized waves with electric field vector forming an angle β with E pass, the ratio of intensities is given by

$$\frac{I}{I_0} = \cos^2(\alpha - \beta) - \sin 2\alpha \sin 2\beta \sin^2 \frac{\phi}{2}. \tag{21.40}$$

For a polarizer–analyzer set such that $(\alpha - \beta) = \pi/2$ the first term vanishes. Choosing the polarizer such that $\alpha = \pi/4$ ($\beta = 3\pi/4$) or $\alpha = 3\pi/4$ ($\beta = \pi/4$) the phase shift ratio of the transmitted to the incident light is

$$\frac{I}{I_0} = \sin^2 \frac{\phi}{2} \tag{21.41}$$

Table 21.2 List of Kerr constants for various liquids ($\lambda = 5890 \text{ \AA}$ and 20°C) (from [21.33, 34])

Liquid	Kerr constant B (10^{-14} m/V^2)	Dielectric constant
Carbon disulfide	3.59	2.6
Carbon tetrachloride	0.0823	2.2
Benzene	0.434	2.3
Acetone	18.13	20.7
Phenyl iso-thiocyanate	101.3	11.5
Nitrobenzene	362.8	36.1
<i>n</i> -Ethyl acetamide	445.1	135
Ethylene carbonate ¹	166.9	89.6
Propylene carbonate ²	111.3	69
<i>o</i> -Dichlorobenzene	47.4	7.5
<i>p</i> -Chlorotoluene	25.6	6.4
Quinoline	16.7	9
Ethyl salicylate	21.8	8.6

¹ 45°C , $\lambda = 5460 \text{ \AA}$

² 23°C , $\lambda = 5460 \text{ \AA}$

with the result that whenever $\phi = 2n\pi$ with $n = 1, 2, \dots$, or equivalently, $E^2 = n/Bl$ we have extinction of the transmitted light. If we monotonously increase the voltage we observe successive extinctions of the light, which

allows us to determine the electric field. This extinction will be global if the electric field is uniform, or local otherwise. This technique has been developed and successfully applied to EHD systems [21.32, 35–37].

21.7 Velocity

The measurement of velocity in electrohydrodynamic flows has special peculiarities that make it more difficult than in hydrodynamic flows. To start with some of the standard techniques, like hot-wire or hot-plate anemometry, are not of use because the probes will not sustain the high electric fields that are always present in EHD flows and would modify the field distribution.

21.7.1 Laser-Doppler Anemometry

In laser-Doppler anemometry a laser beam is split into two beams that are made to interfere inside the liquid. The region of measurement is the volume where the fringes of interference are clearly distinguished. When a particle crosses the fringes it scatters light with a modulated amplitude. The frequency of this modulation is the velocity of the particle divided by the separation between the fringes. A photomultiplier detects the scattered light and an electronic circuit extracts the frequency of the modulation. The signal-to-noise ratio is high if the particle diameter is of the order of the fringe separation, typically of the order of $1\ \mu\text{m}$. For a more rigorous and detailed description see [21.38].

Some successes have been obtained using laser-Doppler anemometry [21.39–41]. However the common practice of seeding the liquid with particles, unavoidable to produce a sufficient amount of scattered light from the volume of measurement, has to be analyzed in detail. Two requirements, closely related, must be fulfilled: the particles should follow the liquid flow, that is, their drift velocity with respect to the liquid must be negligible. On the other hand the seeding particles should not alter the electric field distribution, otherwise their presence will modify the flow structure.

Let us analyze the factors that could induce a drift velocity

1. Common to all hydrodynamic flows is the ability of the particles to respond to a step change in the liquid flow. This is determined by the time scale [21.42]:

$$\tau_1 = \frac{2a^2\rho_p}{9\eta}, \quad (21.42)$$

where a is the radius of the particle, ρ_p its density, and η the liquid viscosity. This time scale is obtained by balancing the viscous and inertial force on an accelerated particle. For particles smaller than $1\ \mu\text{m}$ in size this time is less than $1\ \mu\text{s}$, and therefore does not represent any limitation.

2. If the density of the particles differs from that of the liquid sedimentation takes place. The typical velocity of sedimentation is

$$u = \frac{2(\rho_p - \rho)a^2g}{9\eta} \quad (21.43)$$

which is of order of $1\ \mu\text{m/s}$ for particles in the micrometer range [21.42]. As we have mentioned the velocity of electrohydrodynamic flows is of order of $\sqrt{\epsilon/\rho}E$, where the factor $\sqrt{\epsilon/\rho}$ is called the electrohydrodynamic mobility and is of order $10^{-7}\ \text{m}^2\ \text{V/s}$. For an electric field of order $10^6\ \text{V/m}$ the velocity due to the EHD flow is well above the sedimentation of particles.

3. The major concern for stationary flows is the possible charge of the particles. The motion of charged particles in electric fields is referred to as electrophoresis. The electrophoretic mobility of charged particles is of order of $\epsilon\zeta/\eta$, where ζ is the zeta potential [21.43]. This mobility is to be compared with the electrohydrodynamic mobility. For zeta potentials of the order of $100\ \text{mV}$ and liquids of viscosity $1\ \text{cP}$, the electrophoretic mobility is of order $10^{-9}\ \text{m}^2\ \text{V/s}$, whereas the electrohydrodynamic mobility is of order $10^{-7}\ \text{m}^2\ \text{V/s}$. Although this may not be a problem in regions where the flow is vigorous and well established, it may limit the use of laser-Doppler anemometry in EHD flows near the onset of the flow or in certain regions.
4. If the particles are uncharged, the dielectric force may come into play. In the case of a spherical particle this force is [21.44]

$$F = 2\pi^3\epsilon\ \text{Re}[K(\omega)]\nabla^2 E, \quad (21.44)$$

where $K(\omega)$ is a function of the frequency and is not very different from one. From the balance of

this force and the viscous drag the mobility of the particle is estimated as

$$\frac{a^2 \epsilon E}{\eta l}, \quad (21.45)$$

where l is a typical length scale for the variation of the electric field (typically the distance between the electrodes). Taking 10^6 V/m for the electric field, 1 mm for l and $\eta \approx 10^{-3}$ the dielectric force leads to a mobility of order 10^{-11} m² V/s for a 1 μm particle. Therefore this effect is negligible in the presence of other effects, although it may be noticeable for uncharged particles.

21.8 Visualization

Along with velocimetry, flow visualization is sometimes very useful in determining the flow structure. Let us briefly recall some of the most common techniques of flow visualization used in electrohydrodynamics.

21.8.1 Shadowgraph and Schlieren

In shadowgraph and Schlieren visualization the variation of the index of refraction determines the light pattern. In the shadowgraph technique a parallel beam of light crosses the sample and is observed or photographed on a screen. The differences in index of refraction induce different curvatures of the light rays, thus producing regions of shadow or brightness on the screen. In the Schlieren technique the beam that emanates from the sample is focused by a lens and a knife edge placed at the focal point cuts off half of the beam. This procedure increases the contrast on the screen. Actually in the Schlieren technique the intensity of light on the screen depends on the gradient of the index of refraction of the corresponding zone, whereas in shadowgraph the intensity depends on the second derivative of the index of refraction. For this reason, the Schlieren method is usually preferred (details can be found in [21.46]).

In EHD flows the gradient of the index of refraction has to be induced externally. Usually this is achieved by heating one of the electrodes. Precautions have to be taken in order not to induce noticeable natural convection or electrothermal flows. For natural convection the velocity induced by a temperature difference ΔT may be estimated from the balance between the inertial term

Particles made from PMMA (poly-methyl methacrylate), TiO₂ or glass have been used successfully by various authors. However it is important to remember, especially in the case of PMMA or other latex particles, that the presence of functional groups in the particle surface and of additives in the liquid may drastically change the particle charge.

Besides the use of laser-Doppler anemometry to measure the liquid velocity, this technique has been used with success for the determination of the electrophoretic mobility of colloidal particles [21.45]. Indeed some commercially available apparatuses for evaluating the mobility and size of colloidal particle make use of this technique.

and the buoyancy term in the Navier–Stokes equation:

$$\rho \frac{u^2}{l} \sim \rho \beta \Delta T g \quad (21.46)$$

which yields

$$u = (g \beta l \Delta T)^{1/2}, \quad (21.47)$$

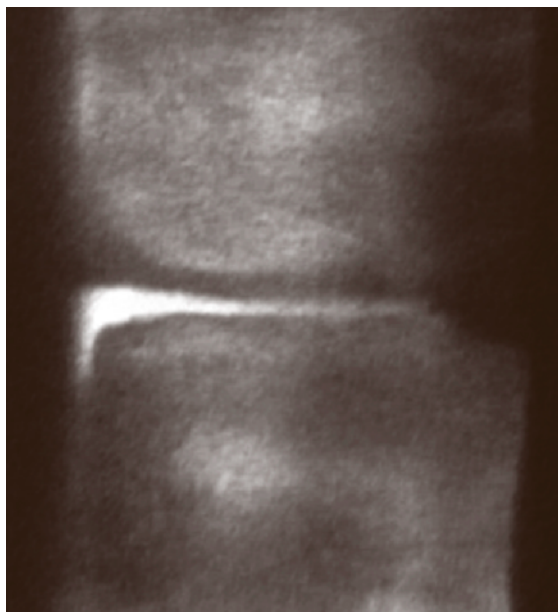


Fig. 21.5 Schlieren photograph of an EHD plume in silicone oil. The distance between the blade and the plane electrodes is 16.4 mm. The applied tension is 24 kV

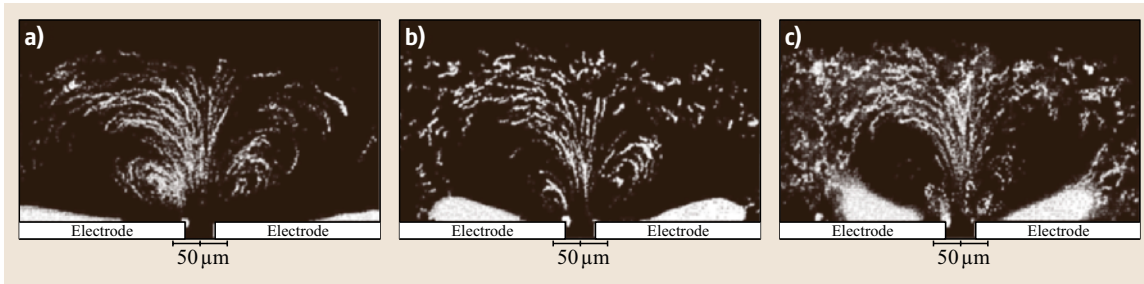


Fig. 21.6 Visualization of an EHD flow between microelectrodes using fluorescent particles (after 21.47)

where β is the coefficient of volumetric thermal expansion. Taking $\beta = 10^{-4} \text{ K}^{-1}$ and $l = 1 \text{ mm}$ the velocity estimated is less than 0.01 m/s for differences of temperature of the order of 1 K . This value is below the expected velocities for EHD flows. Problems arise in large containers, where the velocity induced by natural convection may be non-negligible.

Schlieren techniques have been successful in the study of EHD plumes [21.23, 48]. Figure 21.5 shows a EHD plume originating from a blade at 24 kV [21.23]. The liquid is silicone oil of viscosity $5 \times 10^{-5} \text{ m}^2/\text{s}$ and the distance between the blade and the opposite electrode is 16.8 mm .

21.8.2 Tracer Particles

The use of particles for visualization in EHD flows suffers from the same problems as seeding in laser-Doppler

anemometry, which have already been discussed. However the use of fluorescent particles has opened a new field in flow visualization. The light from fluorescent particles is visible through optical microscopes even at magnifications at which the particles themselves are not visible. This fact allows the use of smaller particles, thus assuring that the particles follow the flow.

Fluorescent particles are typically made from a polymer, such as polystyrene, with a fluorescent dye added. There is a huge variety of fluorescent dyes that excite and emit at different wavelengths. Fluorescent visualization usually requires a microscope with suitable illumination corresponding to the wavelength that excites the dye, and filters that select the wavelength emitted by the particles. Figure 21.6 shows the visualization of flow in a microdevice using latex spheres, 557 nm in diameter, that excite in the blue region of the spectrum and emit green light [21.47].

References

- 21.1 A. Castellanos (Ed.): *Electrohydrodynamics* (Springer, Berlin, Heidelberg 1998), Chaps. 1–4
- 21.2 A. Castellanos, A. González: Nonlinear hydrodynamics of free surfaces, *IEEE T. Dielect. El. In.* **5**, 334–343 (1998)
- 21.3 A. Castellanos: Entropy production and the temperature equation in electrohydrodynamics, *IEEE T. Dielect. El. In.* **10**, 22–26 (2003)
- 21.4 F. Pontiga, A. Castellanos: Physical mechanisms of instability in a liquid layer subjected to an electric field and a thermal gradient, *Phys. Fluids A* **6**, 1684–1701 (1994)
- 21.5 J.R. Melcher: *Continuum Electromechanics* (MIT Press, Cambridge 1981)
- 21.6 J.M. Crowley: Dimensionless ratios in electrohydrodynamics. In: *Handbook of Electrostatics*, ed. by J.S. Chang, J.M. Crowley, A.J. Kelly (Marcel-Dekker, New York 1995) pp. 99–119
- 21.7 A. Khayari, A.T. Pérez: Nonlinear dynamics of a bouncing ball driven by electric forces, *Int. J. Bifurcat. Chaos* **13**(10), 2959–2975 (2003)
- 21.8 A. Ramos, H. González, A. Castellanos: Experiments on dielectric liquid bridges subjected to electric fields, *Phys. Fluids A* **6**, 3206–3208 (1994)
- 21.9 J.R. Melcher: *Field-Coupled Surface Waves* (MIT Press, Cambridge, MA 1963)
- 21.10 L. Lobry, E. Lemaire: Viscosity decrease induced by a DC electric field in a suspension, *J. Electrostat.* **47**(1–2), 61–69 (1999)
- 21.11 E. Lemaire, L. Lobry: Reverse electrorheological effect: A suspension of colloidal motors, *Int. J. Mod. Phys. B* **15**(6–7), 780–787 (2001)
- 21.12 E. Lemaire, L. Lobry: Chaotic behavior in electro-rotation, *Physica A* **314**(1–4), 663–671 (2002)
- 21.13 A. Castellanos, A. Ramos, A. González, N.G. Green, H. Morgan: Electrohydrodynamics and dielectrophoresis in microsystems: scaling laws, *J. Phys. D* **36**, 2584–2597 (2003)

- 21.14 P.K. Watson: Space charged limited currents in liquid dielectrics. In: *Electrohydrodynamics*, ed. by A. Castellanos (Springer, Berlin, Heidelberg 1998), Chap. 10
- 21.15 A. Denat, B. Gosse, J.P. Gosse: Electrical conduction of solutions of an ionic surfactant in hydrocarbons, *J. Electrostat.* **12**, 197–205 (1982)
- 21.16 L. Onsager: Deviations from Ohm's law in weak electrolytes, *J. Chem. Phys.* **2**, 599–615 (1934), in *The collected works of Lars Onsager* (World Scientific, Singapore 1996)
- 21.17 P.K. Watson, T.M. Clancy: Electron injection technique for investigating processes in insulating liquids and solids, *Rev. Sci. Instrum.* **36**(2), 217–222 (1965)
- 21.18 F. Vega, A.T. Pérez: Corona-induced electrohydrodynamic instabilities in low conducting liquids, *Exp. Fluids* **34**, 726–735 (2003)
- 21.19 R. Tobazéon: Sur l'electrodialyse de liquides non polaires ou faiblement polaires, *C. R. Acad. Sci.* **282**, C153–C156 (1976)
- 21.20 N.J. Félici, R. Tobazéon: Charge carrier elimination and production by electrodyalitic polymers in contact with dielectric liquids, *J. Electrostat.* **11**, 135–161 (1981)
- 21.21 A. Denat, B. Gosse, J.P. Gosse: Ion injections in hydrocarbons, *J. Electrostat.* **7**, 220–225 (1979)
- 21.22 A.I. Zhakin: Classic theories of ion recombination and dissociation in liquids. In: *Electrohydrodynamics*, ed. by A. Castellanos (Springer, Berlin, Heidelberg 1998), Chap. 6
- 21.23 A.T. Pérez, P.A. Vázquez, A. Castellanos: Dynamics and linear stability of charged jets in dielectric liquids, *IEEE T. Ind. Appl.* **31**(4), 761–767 (1995)
- 21.24 F.M.J. McCluskey, P. Atten, A.T. Pérez: Heat Transfer enhancement by electroconvection resulting from an injected space charge between parallel plates, *Int. J. Heat Mass Tran.* **34**(9), 2237–2250 (1991)
- 21.25 M. Medrano, A.T. Pérez, C. Soria-Hoyo: Design of a conductivity meter for highly insulating liquids, *J. Phys. D: Appl. Phys.* **40**, 1477–1482 (2007)
- 21.26 M. Hilaire, C. Marteau, R. Tobazéon: Apparatus developed for measurement of the resistivity of highly insulating liquids, *IEEE T. Electr. Insul.* **23**(4), 779–787 (1988)
- 21.27 R. Tobazéon, J.C. Filippini, C. Marteau: On the measurement of the conductivity of highly insulating liquids, *IEEE T. Dielect. El. In.* **1**(6), 1000–1004 (1994)
- 21.28 P. Atten, J.P. Gosse: Transient of one-carrier injections in polar liquids, *J. Chem. Phys.* **51**(7), 2804–2811 (1969)
- 21.29 H.T. Davis, S.A. Rice, L. Meyer: On the kinetic theory of simple dense fluids – XI. Experimental and theoretical studies of positive ion mobility in liquid Ar, Kr, and Xe, *J. Chem. Phys.* **37**(5), 947–956 (1962)
- 21.30 B.L. Henson: Mobility of positive ions in liquefied argon and nitrogen, *Phys. Rev.* **135**(4A), A1002–A1008 (1964)
- 21.31 A. Alj, J.P. Gosse, B. Gosse, A. Denat, M. Nemancha: Influence de la nature du surfactant ionique sur la conduction électrique de ses solutions dans le cyclohexane, *Rev. Phys. Appl.* **22**, 1043–1053 (1987)
- 21.32 A. Denat: *Etude de la conduction électrique dans les solvants non polaires*, Ph.D. Thesis (Université Joseph-Fourier, Grenoble 1982)
- 21.33 D.E. Gray (Ed.): *American Institute of Physics Handbook* (McGraw-Hill, New York 1957)
- 21.34 J.C. Lacroix: *Etude et réalisation d'une cellule de Kerr a champ électrique tournant*, Ph.D. Thesis (Université Joseph-Fourier, Grenoble 1970)
- 21.35 R. Tobazéon: *Etude du transfert convectif de charges électriques par un jet de liquide isolant et application a la generation de tensions élevés*, Ph.D. Thesis (Université Joseph-Fourier, Grenoble 1973)
- 21.36 M. Zahn: Optical, electrical and electromechanical measurement methodologies of field, charge and polarization in dielectrics, *IEEE T. Dielect. El. In.* **5**, 627–650 (1998)
- 21.37 D. Filipovic, P. Osmokrovic, Z. Lazarevic: Electro-optical Kerr effect in liquid dielectrics, *Mater. Sci. Forum* **413**, 197–200 (2002)
- 21.38 L.E. Drain: *The Laser Doppler Technique* (Wiley, New York 1980)
- 21.39 R. DisseInkötter, K. Barner: Seeding of electrohydrodynamic convection flows for light scattering experiments, *J. Electrostat.* **19**, 323–336 (1987)
- 21.40 F.M.J. McCluskey, A.T. Pérez: The electrohydrodynamic plume between a line source of ions and a flat plate, *IEEE T. Electr. Insul.* **27**(2), 334–341 (1992)
- 21.41 P. Atten, M. Haidara: Electrical conduction and EHD motion of dielectric liquids in a knife-plane electrode assembly, *IEEE T. Electr. Insul.* **20**(2), 187–198 (1985)
- 21.42 R.J. Adrian: Laser velocimetry. In: *Fluid Mechanics Measurements*, ed. by R.J. Goldstein (Springer, Berlin, Heidelberg 1983)
- 21.43 R.J. Hunter: *Foundations of Colloid Science*, 2nd edn. (Oxford Univ. Press, Oxford 2001)
- 21.44 T.B. Jones: *Electromechanics of Particles* (Cambridge Univ. Press, New York 1991)
- 21.45 J.P. Dalbiez, K. Tabti, P.J. Derian, M. Drifford: Vélométrie Doppler sous champ électrique: technique et application à l'étude de la mobilité électrophorétique des colloïdes et des polyélectrolytes, *Rev. Phys. Appl.* **22**, 1013–1024 (1987)
- 21.46 R.J. Goldstein: Optical systems for flow measurement: Shadowgraph, Schlieren and interferometric techniques. In: *Fluid Mechanics Measurements*, ed. by R.J. Goldstein (Springer, Berlin, Heidelberg 1983)
- 21.47 N.G. Green, A. Ramos, A. González, H. Morgan, A. Castellanos: Fluid flow induced by nonuniform

- AC electric fields in electrolytes on microelectrodes – III. Observation of streamlines and numerical simulation, Phys. Rev. E **66**, 026305–026315 (2002)
- 21.48 B. Malraison, P. Atten, A.T. Pérez: Panaches chargés résultant de l'injection d'ions dans un liquide isolant par une lame ou une pointe placée en face d'un plan, J. Phys. III **4**, 75–85 (1994)