

# A Short Course on Turbulence

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## A SHORT COURSE ON TURBULENCE

September 2010

Instructor:	John Kim, UCLA, jkim@seas.ucla.edu
Class Hours:	MW, 9-11 am
Office Hours:	By appointment
Textbook:	Turbulent Flows by Pope, Cambridge University Press; See also the reference list below.
Prerequisites:	Knowledge of graduate-level fluid mechanics and applied mathematics.
Grading:	There will be a short project assignment, and details will be announced in class.

### Additional Notes:

1. Class attendance is not mandatory. However, if you decide to attend any particular lecture, I request you to come **on time**. We will start at **9:05 am**. If you decide to skip a class, it is your responsibility to obtain information regarding class handouts and other announcements made available during the class. Please contact your colleagues for handouts and class materials, etc.
2. There will be no formal homework assignments for this course. I may suggest useful exercises during lectures and it is recommended that you carry out these exercises. If you need help on these suggested exercises, I will be glad to help you out. You do not need to turn in these exercises, and hence, they will not be counted toward your final grade. There will be a short project, which will require your presentation. I will explain more about this project in class.
3. I have an open-door policy for office hours. You can stop by my office any time. Unless I am occupied with an urgent matter, I will be available. If you prefer, you can send me e-mail, and I will try to respond promptly. I need your feedback to adjust the scope of this course as I go along.

## REFERENCES:

Pope, Turbulent Flows, Cambridge Univ. Press (2000):

Our primary textbook. New and very thorough. Written by a currently active researcher (a good friend of mine!) with modern views on turbulence.

Tennekes and Lumley, A First Course in Turbulence, MIT Press (1972):

A popular textbook, but don't get fooled by its title. If you have some prior knowledge on turbulence, this is a great book. However, it is a bit terse for beginners.

Hinze, Turbulence, 2<sup>nd</sup> edition, McGraw-Hill (1975):

A classic good old reference book.

Townsend, The Structure of Turbulent Shear Flow., Cambridge Univ. Press (1976):

A good reference for turbulent flows; every student interested in turbulent flows must own a copy (available in paperback!).

Batchelor, The Theory of Homogeneous Turbulence, Cambridge Univ. Press (1953):

A very focused monograph on the fundamentals of turbulence.

Monin and Yaglom, Statistical Fluid Mechanics, MIT Press (1971):

Two-volume compilation by the famous Russian scientists.

Kim and Leal (eds.), Physics of Fluids, American Institute of Physics:

One of the two premier journals for fluid mechanics including turbulence. We will discuss some papers published in this journal.

Worster (ed.), Journal of Fluid Mechanics, Cambridge University Press:

The other premier journal for fluid mechanics. We also use papers published in this journal.

Davis and Moin (eds.), Annual Review of Fluid Mechanics, Annual Reviews:

A good place to start your literature survey.

# TENTATIVE COURSE OUTLINE

## I Introduction

- I.1 Characteristics of turbulent flows
- I.2 Review of Cartesian tensors

## II The Governing Equations

## III Statistical Description of Turbulent Flows

- III.1 Random variables and probability distributions
- III.2 Random processes and frequency spectra
- III.3 Random fields, statistical stationarity and statistical homogeneity

## IV Mean-Flow Equations

- IV.1 Reynolds decomposition and Reynolds stresses
- IV.2 Turbulence kinetic energy equation and energy budget

## V Scales of Turbulent Motion

- V.1 Energy cascade and Kolmogorov hypotheses
- V.2 Integral, Taylor micro, and Kolmogorov scales
- V.3 Fourier modes and velocity spectra

## VI Wall-Bounded Turbulent Flows

- VI.1 The law of the wall, the velocity-defect law and the log layer
- VI.2 Fully developed turbulent channel flow
- VI.3 Turbulent boundary layer (TBL)
- VI.4 Organized structures

## VII Numerical Simulations

- VII.1 Direct numerical simulation (DNS)
- VII.2 Large-eddy simulation (LES)
- VII.3 Detached-eddy simulation (DES)

# What is turbulence?

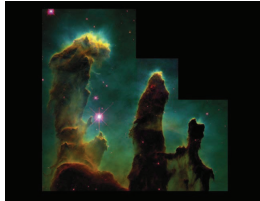
- Sir Horace Lamb (quote from Goldstein in ARFM, **1**, 1969)
  - I am an old man now, and when I die and go to heaven there are two matters on which I hope enlightenment. One is quantum electro-dynamics and the other is turbulence. About the former, I am really rather optimistic.
- Richard Feynman (quote from Lumley in POF, **4**, 1992)
  - Turbulence is the last great unsolved problem of classical physics.
- Anonymous
  - Turbulence is like pornography. It is hard to define, but if you see it, you recognize it immediately.

# Turbulent Flows

M100 galaxy,  $L \sim 10^{23}$ m



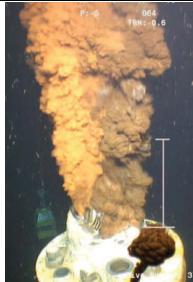
Eagle nebula,  $L \sim 10^{18}$ m



Earth's atmosphere,  $L \sim 10^7$ m



Volcano in Iceland,  $L \sim 10^3$ m

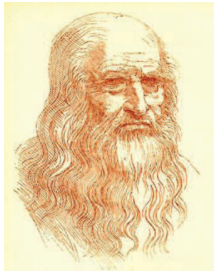


BP oil leak,  $L \sim 10^0$ m



Soap film,  $L \sim 10^{-1}$ m

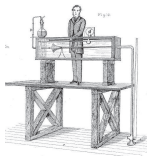
# Leonardo da Vinci (1452-1519)



Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion.

Piomelli's translation [from Lumley, POF A 4(2), 1992]]

# Osborne Reynolds (1842-1912)



(1.) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube, fig. 3.

Fig. 3.



(2.) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3.) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in fig. 4.

Fig. 4.



Any increase in the velocity caused the point of break down to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in fig. 5.

Fig. 5.



“An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels,” *Phil. Trans. Roy. Soc. Lon.* 174



# Reynolds Decomposition

- Decompose into mean and fluctuation:

$$\begin{aligned}u_i &= U_i + u'_i \\ \overline{u_i} &= U_i, \quad \overline{u'_i} = 0\end{aligned}$$

- Reynolds-averaged Navier-Stokes equations (RANS):

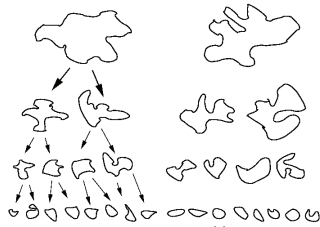
$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} - \overline{u'_i u'_j} \right)$$

- Eddy viscosity, Boussinesq (1842-1929):



$$\overline{u'_i u'_j} = \nu_T \frac{\partial U_i}{\partial x_j}$$

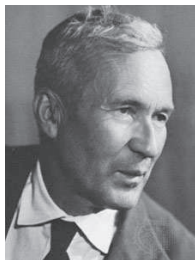
# Energy Cascade: Richardson's (1881-1953) Poem



Richardson (1920):

*Big whorls have little whorls,  
That feed on their velocity;  
And little whorls have lesser whorls,  
And so on to viscosity.*

# Kolmogorov (1903-1987) Hypotheses



- **Local isotropy:** At sufficiently high Reynolds number, the small-scale turbulence motions are statistically isotropic.
- **First similarity:** In every turbulent flow at sufficiently high Reynolds number, the statistic of the small-scale motions have a universal form that is uniquely determined by  $\nu$  and  $\epsilon$ .
- **Second similarity:** In every turbulent flow at sufficiently high Reynolds number, the statistics of the motions in the inertial range have a universal form that is uniquely determined by  $\epsilon$  independent of  $\nu$ .

## Other Giants (deceased ones only)

- T. von Kármán (1881-1963)
- G. I. Taylor (1886-1975)
- L. D. Landau (1908-1968)
- S. Corrsin (1920-1986)
- G. K. Batchelor (1920-2000)
- A. Yaglom (1921-2007)
- R. H. Kraichnan (1928-2008)
- P. G. Saffman (1931-2008)
- Kline, Reynolds, ...

# Project: Images of Turbulent Flows

- Take or collect interesting images of turbulent flows, and explain them using the knowledge you have learned from this class:
  - September 27, Monday, about 5 min. each
- Examples from the last class I taught at UCLA

## Reynolds Number Comparison



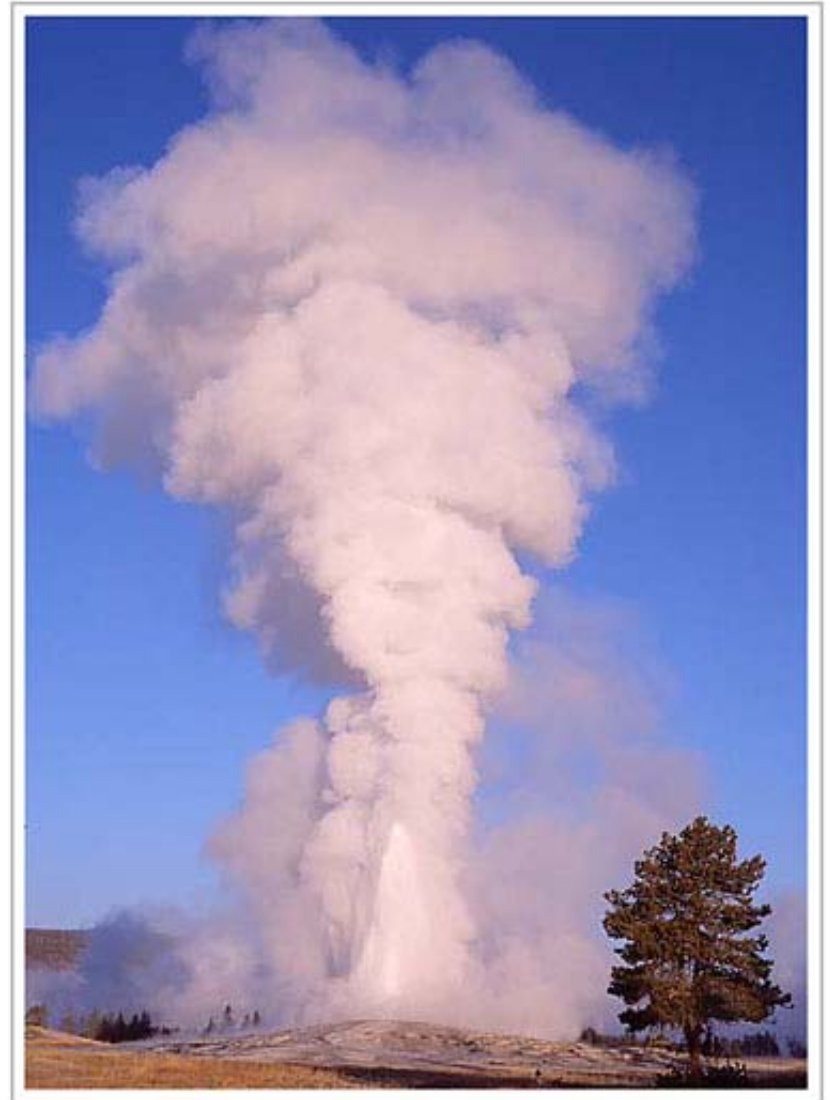
Source: [http://news.bbc.co.uk/media/images/39967000/jpg/\\_39967629\\_whirlpool300300.jpg](http://news.bbc.co.uk/media/images/39967000/jpg/_39967629_whirlpool300300.jpg)

Source: <http://www.tqny.org/NYC062695/HurricaneIsabel10-17-03.jpg>

## Old Faithful ~ Axi-symmetric Jet



Source: <http://photos.case.edu/photo/879.jpg>



Source: <http://www.dannyburk.com/images/old-faithful.jpg>











4

3

2

1

# Turbulence



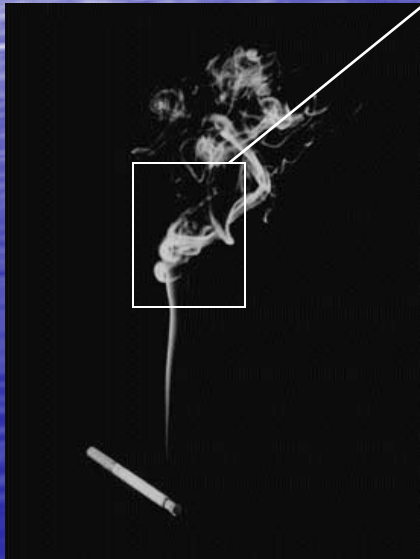
Energy Cascade:

Energy dissipation (viscous forces dominate => low Re)

Energy transferred (energy breakup/transport, decreasing Re)

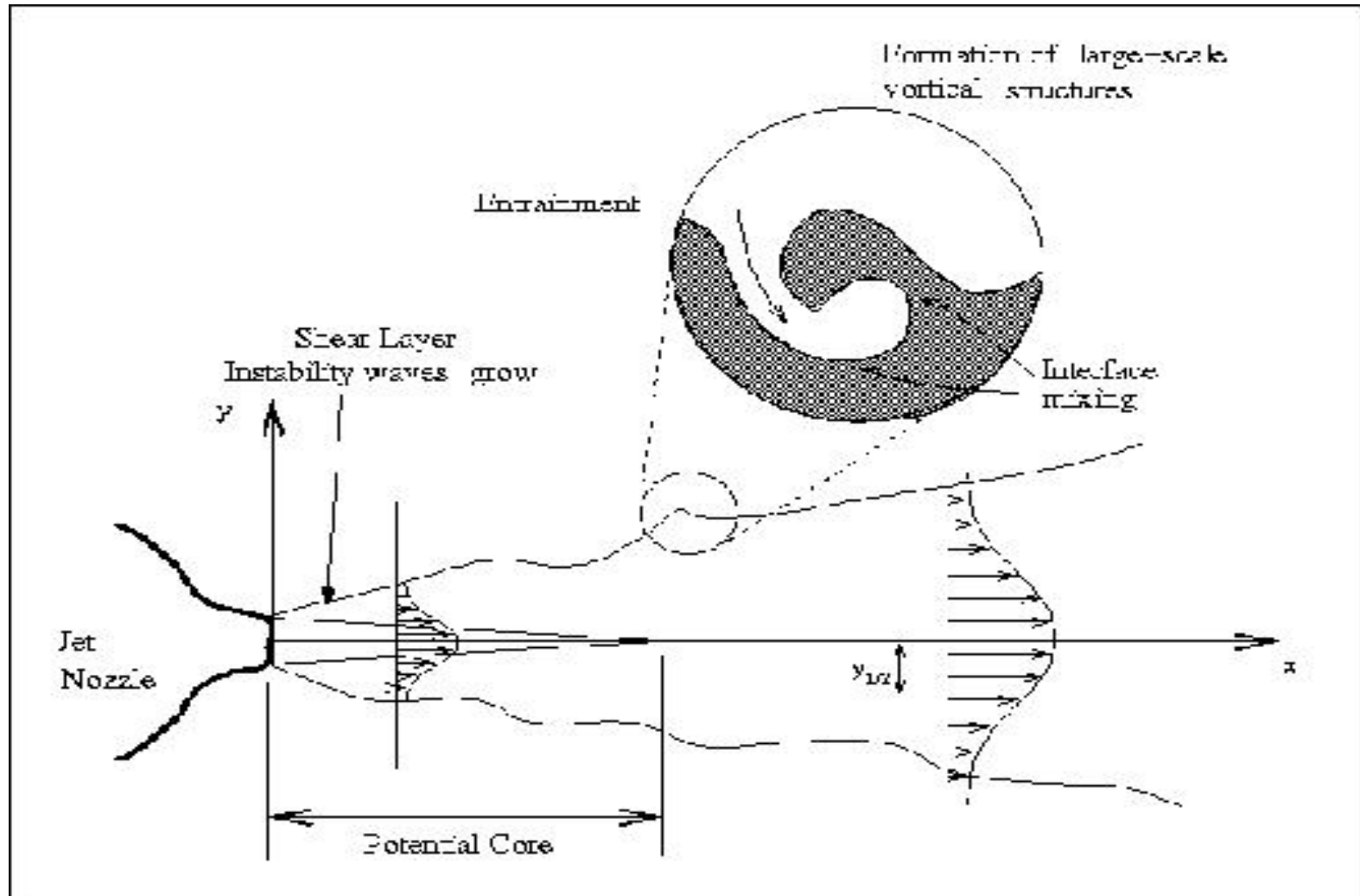
Production of turbulence by mean flow (relative large length scale => high Re)

Re ~ 1000

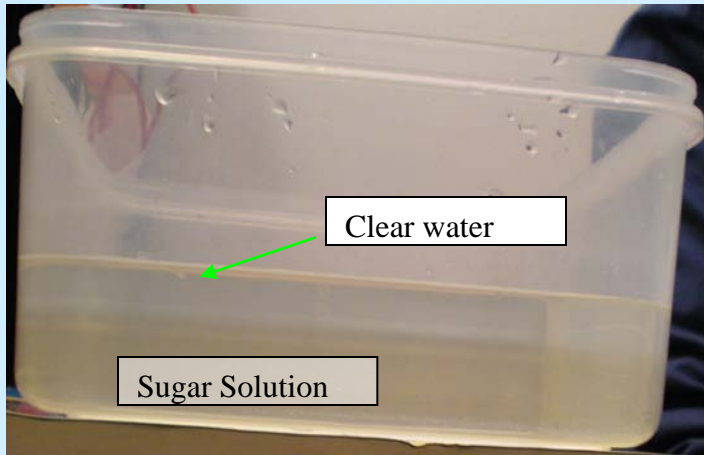




# Concept of Jet



# Images for Quasi 2-D turbulence



1(a) Stratified solution



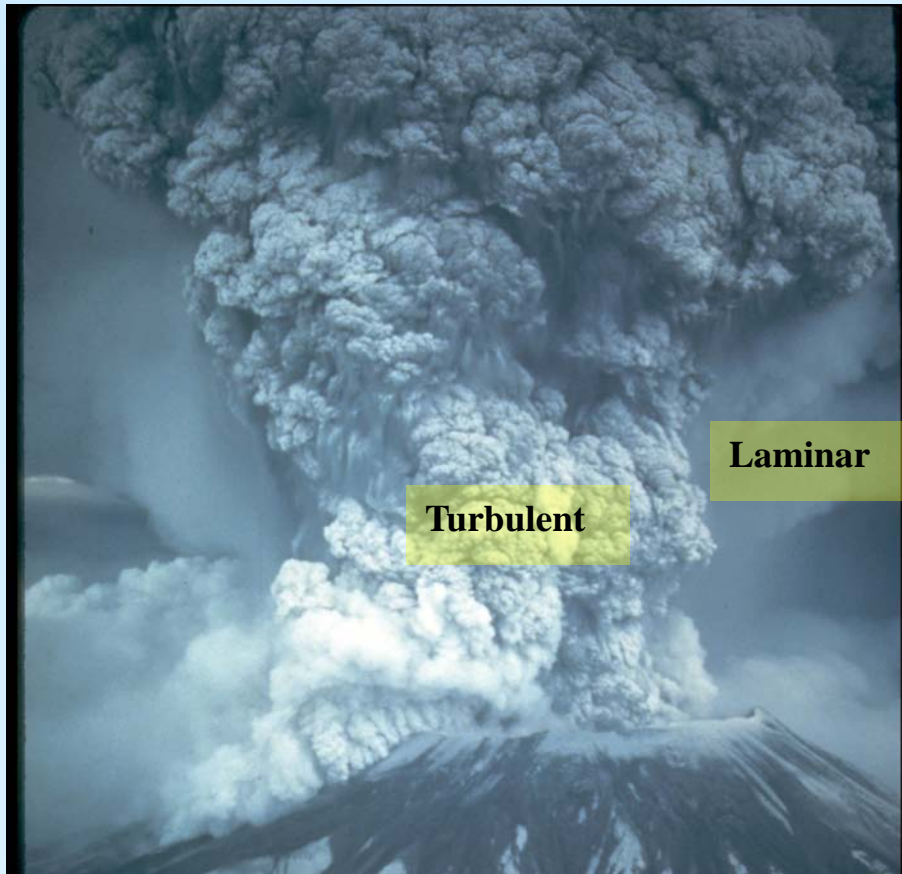
1(b) Reaction to forced turbulence: [Click to play](#)



1(c) At  $t = 5\text{min}$  after disturbance

Small Scale vortices caused by forced turbulence merge to form slowly moving large vortices: Inverse Cascade of energy

# Images for 3-D turbulence



2(a) Volcanic plume from Mt. St. Helens

([http://vulcan.wr.usgs.gov/Volcanoes/MSH/Images/may18\\_images.html](http://vulcan.wr.usgs.gov/Volcanoes/MSH/Images/may18_images.html))

$$\text{Re}_l = \frac{ul}{\nu} \approx 10^8$$



2(b) Plume of turmeric powder mixing in clear water.

$$\text{Re}_l = \frac{ul}{\nu} \approx 10^4$$

- Large difference between scales for Volcanic plume.
- 3-D turbulence energy cascades from large scales to small scales.



# Line Vortices

- Created at wing-tips due to pressure difference between top and bottom of wing



NASA



New York Times

# Crow Instability

- Crow Instability defined by symmetric oscillations of the line vortices
- Instability typically caused by isotropic air turbulence



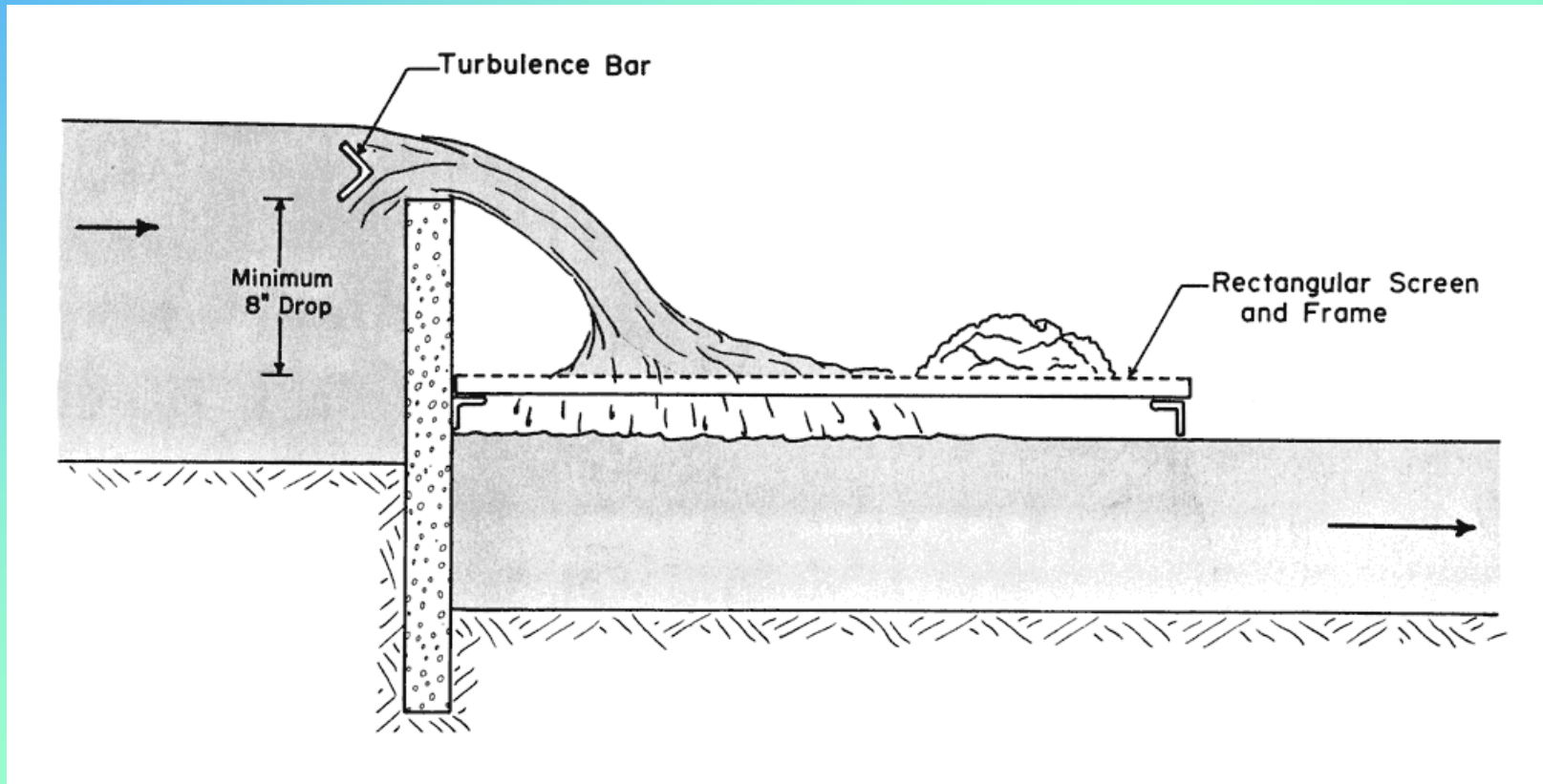
NASA

# Crow Instability

- Oscillations may grow until line vortices merge and form vortex rings



# Self-Cleaning Ditchwater Screen



USDA – Agricultural Research Service

# Horizontal Screen



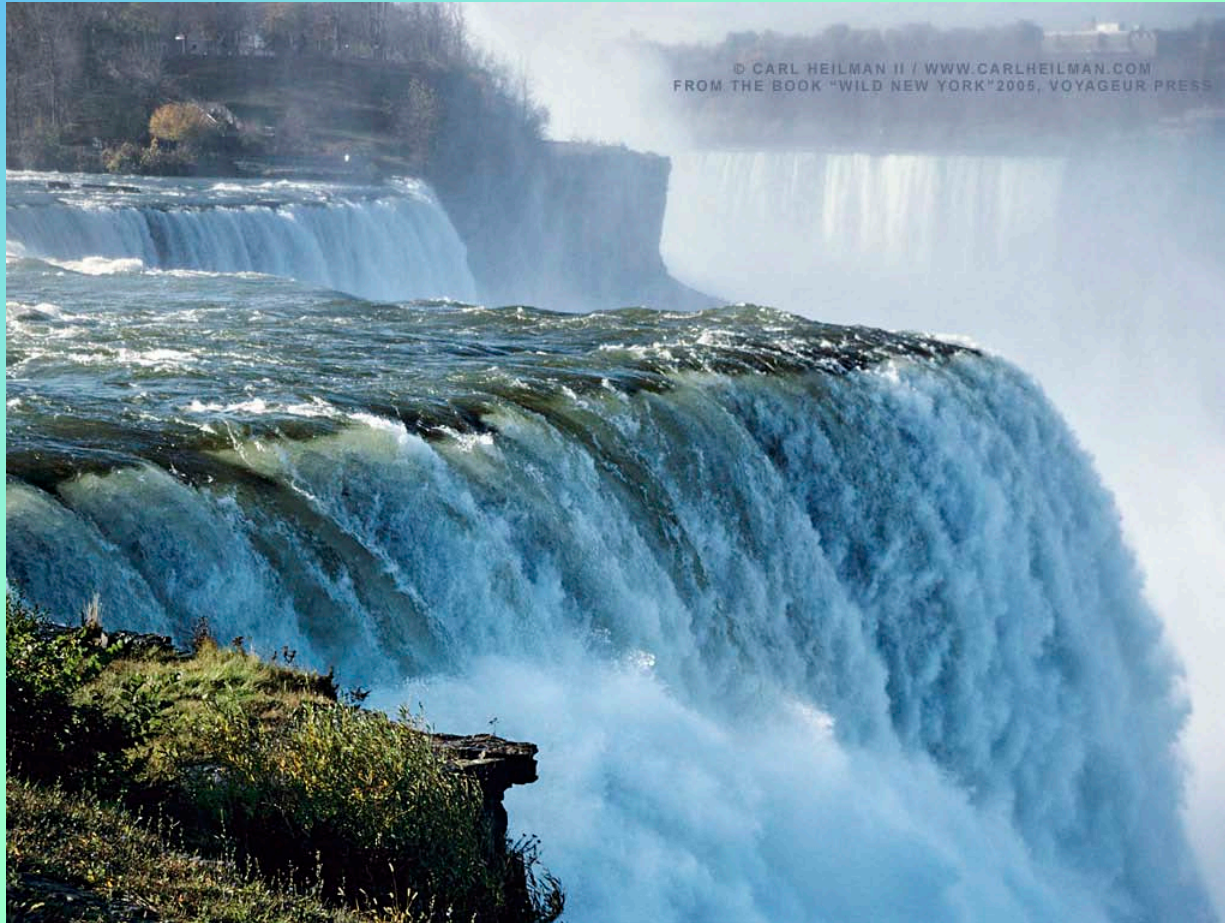
# Horizontal Screen



# Horizontal Screen



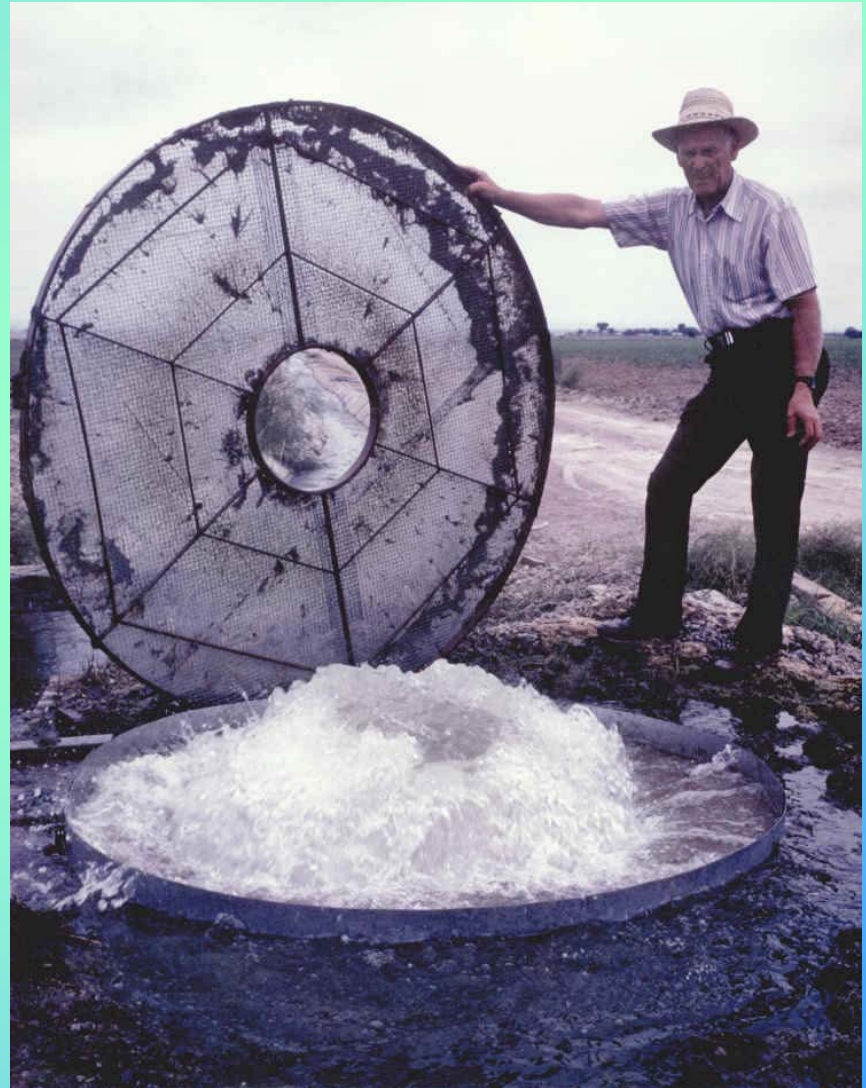
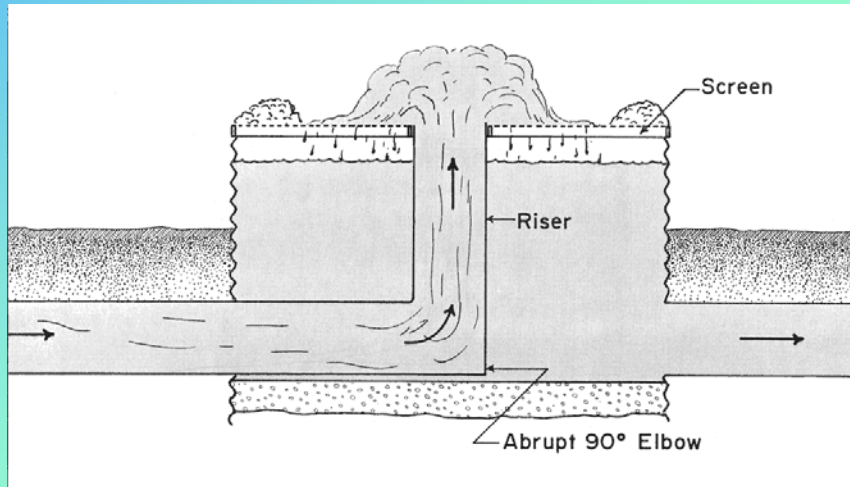
# Higher Re



Niagara Falls



# Fountain 'Bubbler' Screen for Piped Water



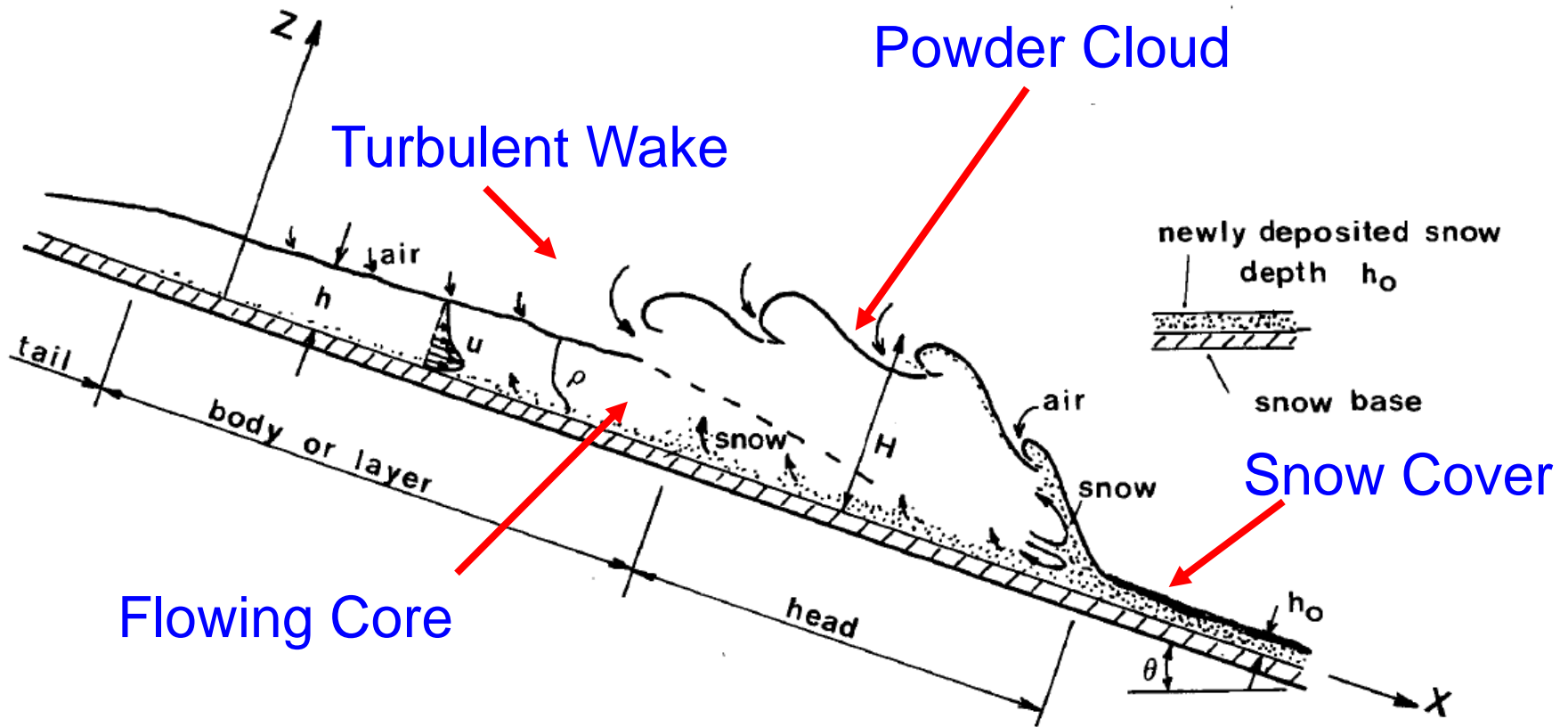
# Higher Re





Denali National Park, Alaska

# Snow-Powder Avalanche



Sketch of snow and air entrainment mechanisms

(Hopfinger, ARFM 15:47-76, 1983)





# Cloud Vortex



- ❖ Example of atmospheric turbulence caused by strong natural convection
- ❖ Eddies with various scales were observed
- ❖ the smallest scale eddy obeys the isotropic approximation
- ❖ the size of the largest scale eddy decrease as the cloud get closer and closer to the ground

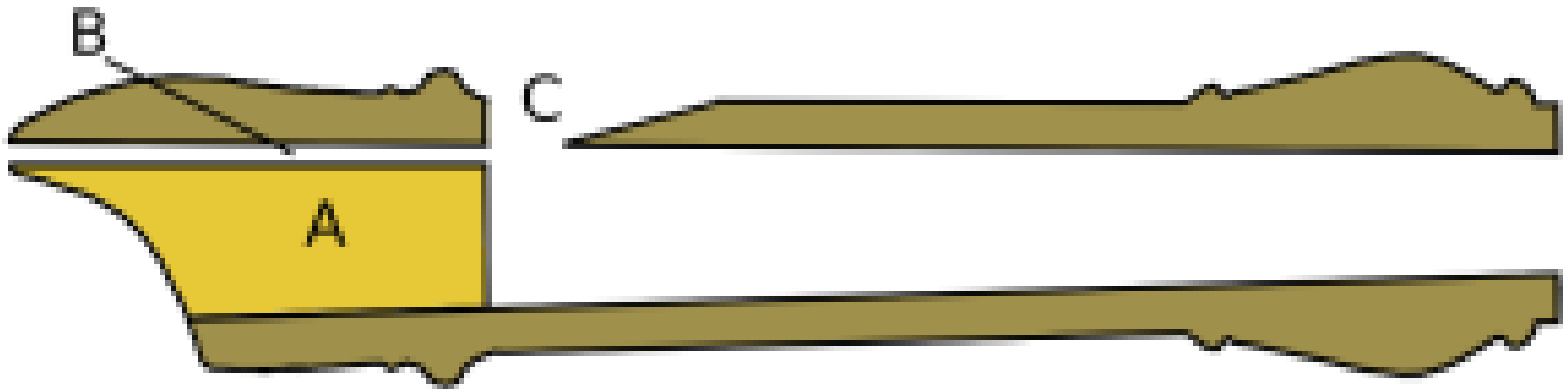
Rough Estimation based upon Kolmogorov model:

Assume the  $Re \sim 10^6 - 10^7$

$$\frac{\eta}{l_0} = Re^{-\frac{3}{4}} \quad \Rightarrow \quad \eta \approx 3 \times 10^{-5} l_0 \text{ to } 5 \times 10^{-6} l_0$$

NASA photo of a cloud vortex over the Madeira Islands.

# A Diagram of a Recorder





# Vortex and Turbulent Boundary Layer



# Karman Vortex Street

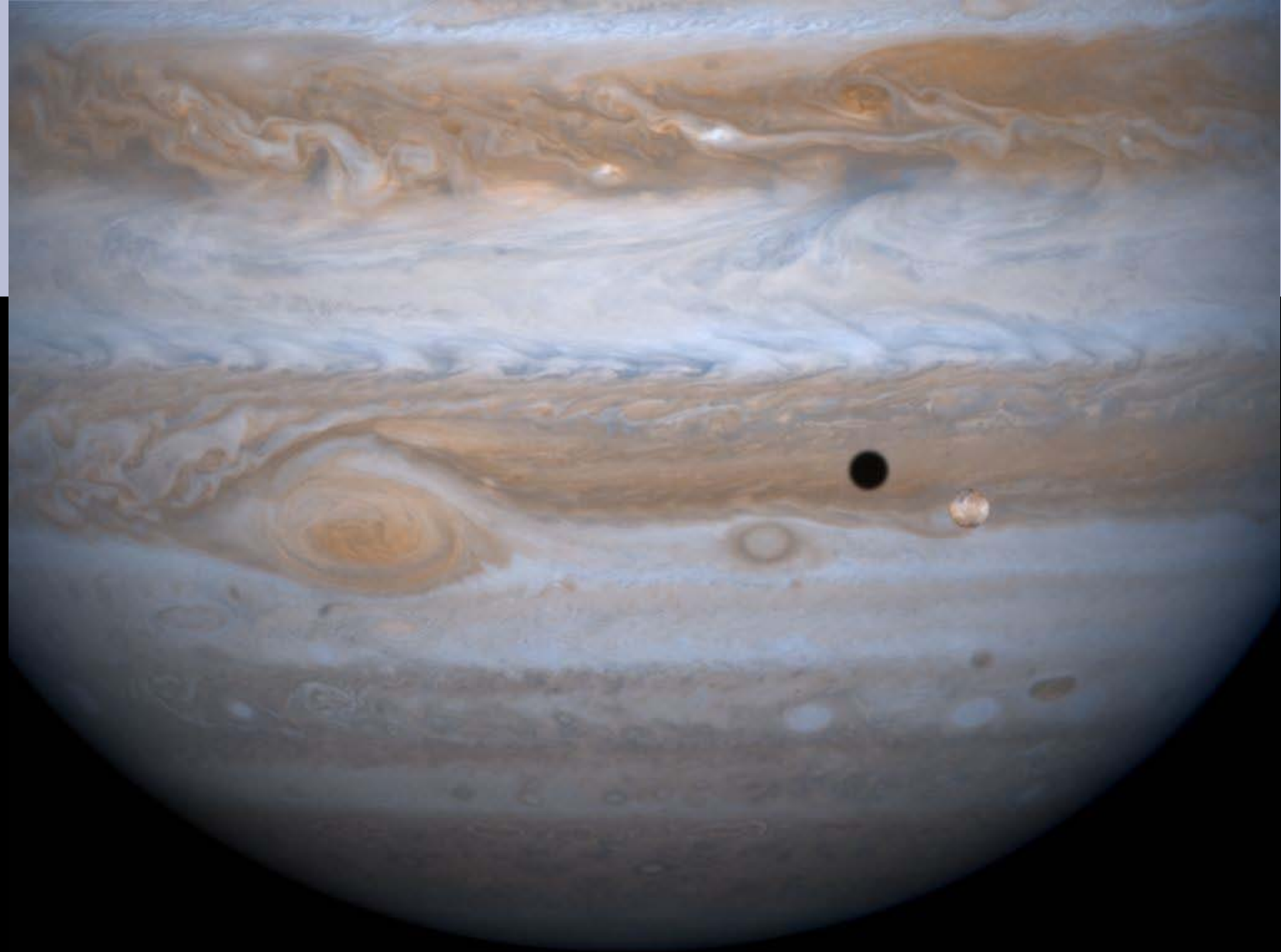


# Wall of Fire



# An Explosion

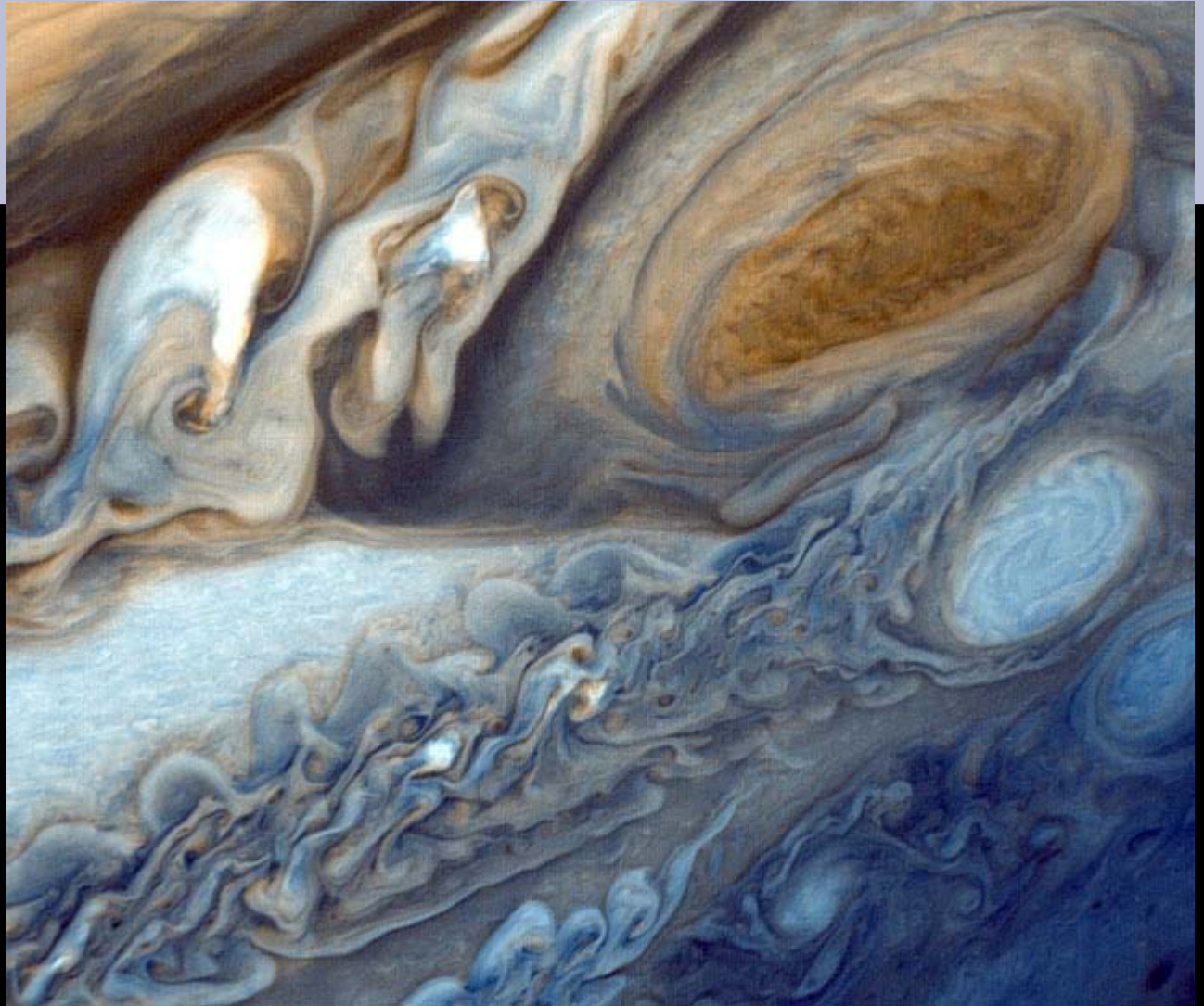




# Atmospheric Turbulence: Jupiter

Tristan Sharp  
MAE 252  
5/13/08

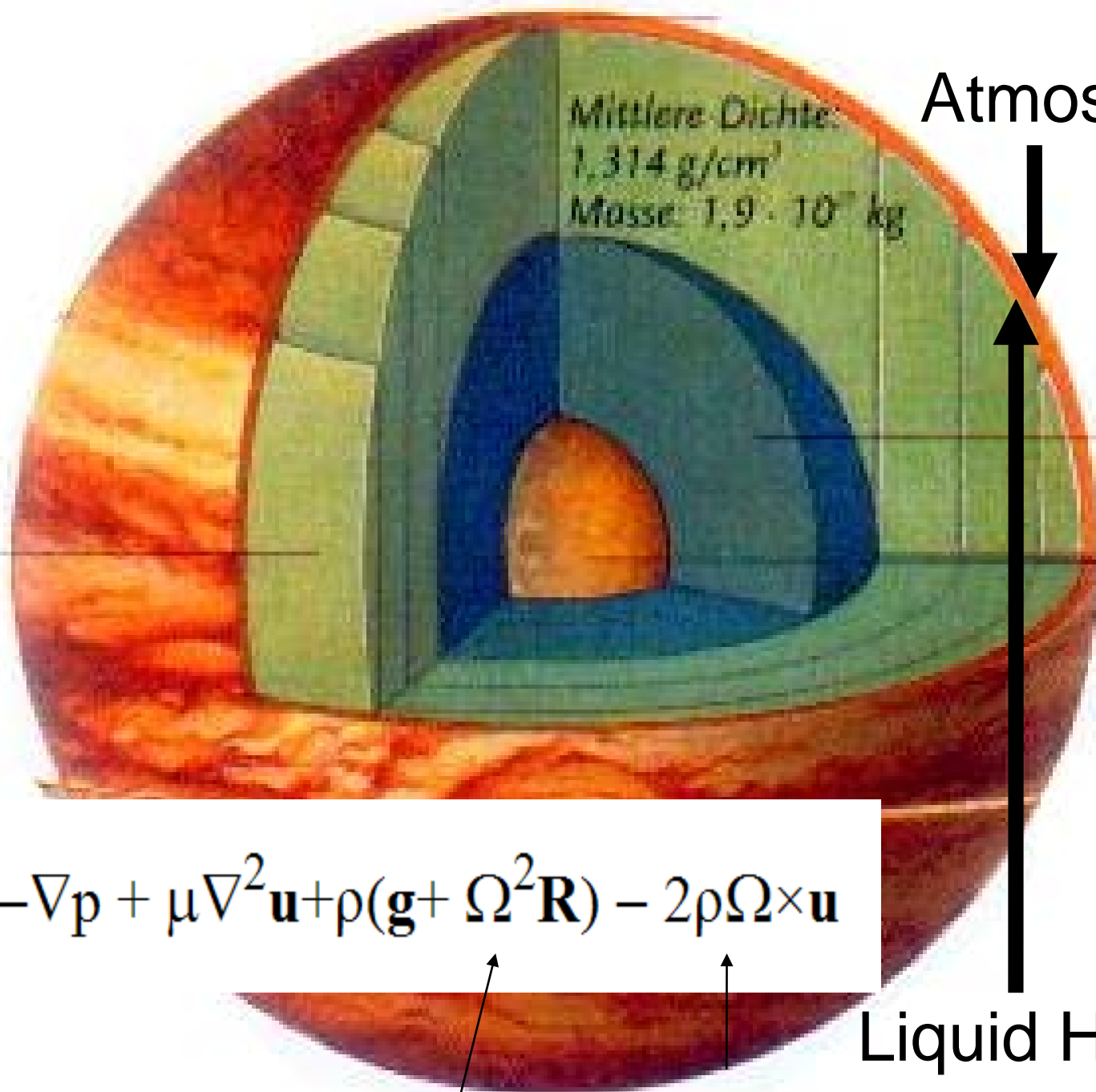
$$\begin{aligned} \text{Re}_T &= U * d * \rho / \mu \\ &\sim 10 \text{ m/s} * \\ &10^7 \text{ m} * \\ &10^{-1} \text{ kg/m}^3 / \\ &5 \cdot 10^{-6} \text{ kg / m s} \\ &> 10^{12} \end{aligned}$$



Flow information: *Jupiter: The Planet, Satellites and Magnetosphere*  
By Fran Bagenal

Viscosity calculator

<http://www.lmnoeng.com/Flow/GasViscosity.htm>



Mittlere Dichte:  
 1,314 g/cm<sup>3</sup>  
 Masse: 1,9 · 10<sup>27</sup> kg

Atmosphäre

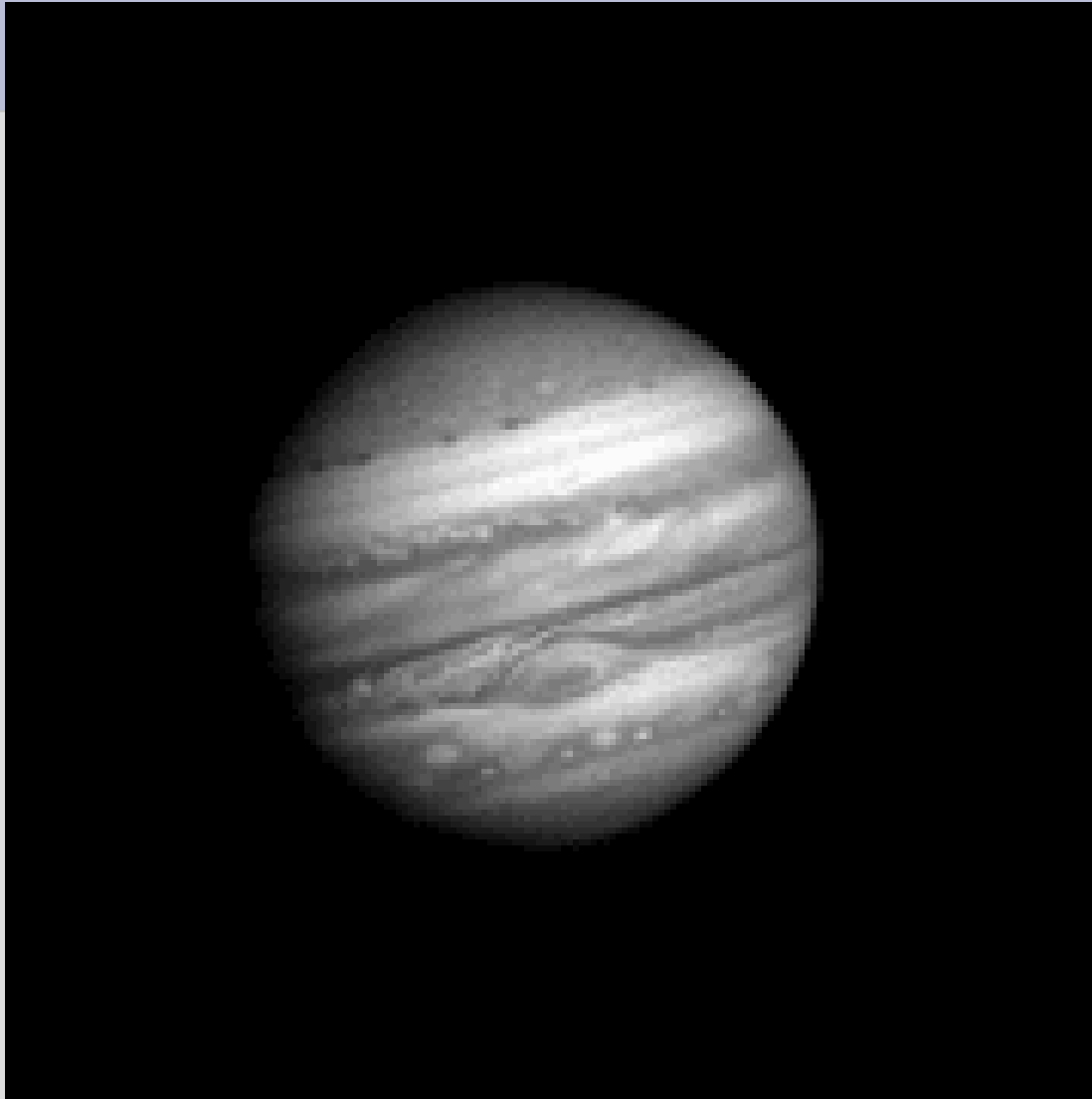


Liquid Hydrogen

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho(\mathbf{g} + \Omega^2 \mathbf{R}) - 2\rho \Omega \times \mathbf{u}$$

Centrifugal

Coriolis Effect



Voyager II

3 months

resolution 300 – 60 km

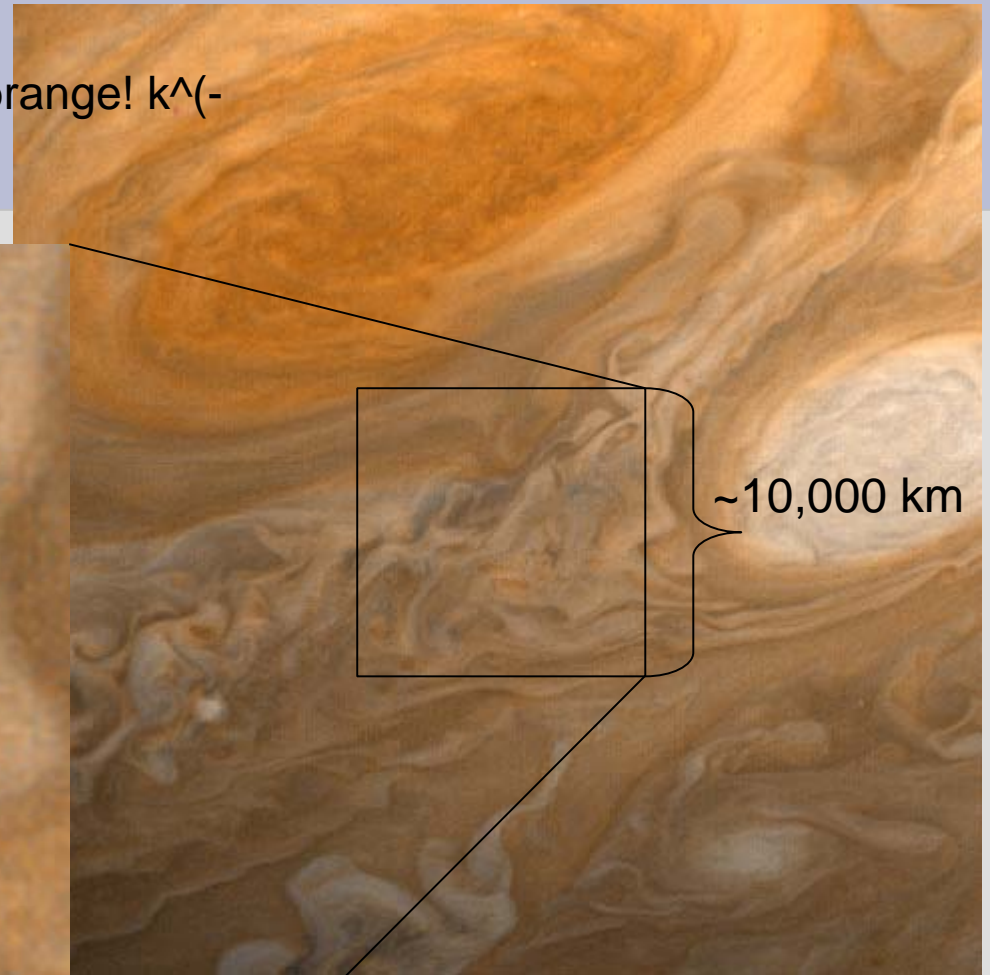
[www.nasa.gov](http://www.nasa.gov)



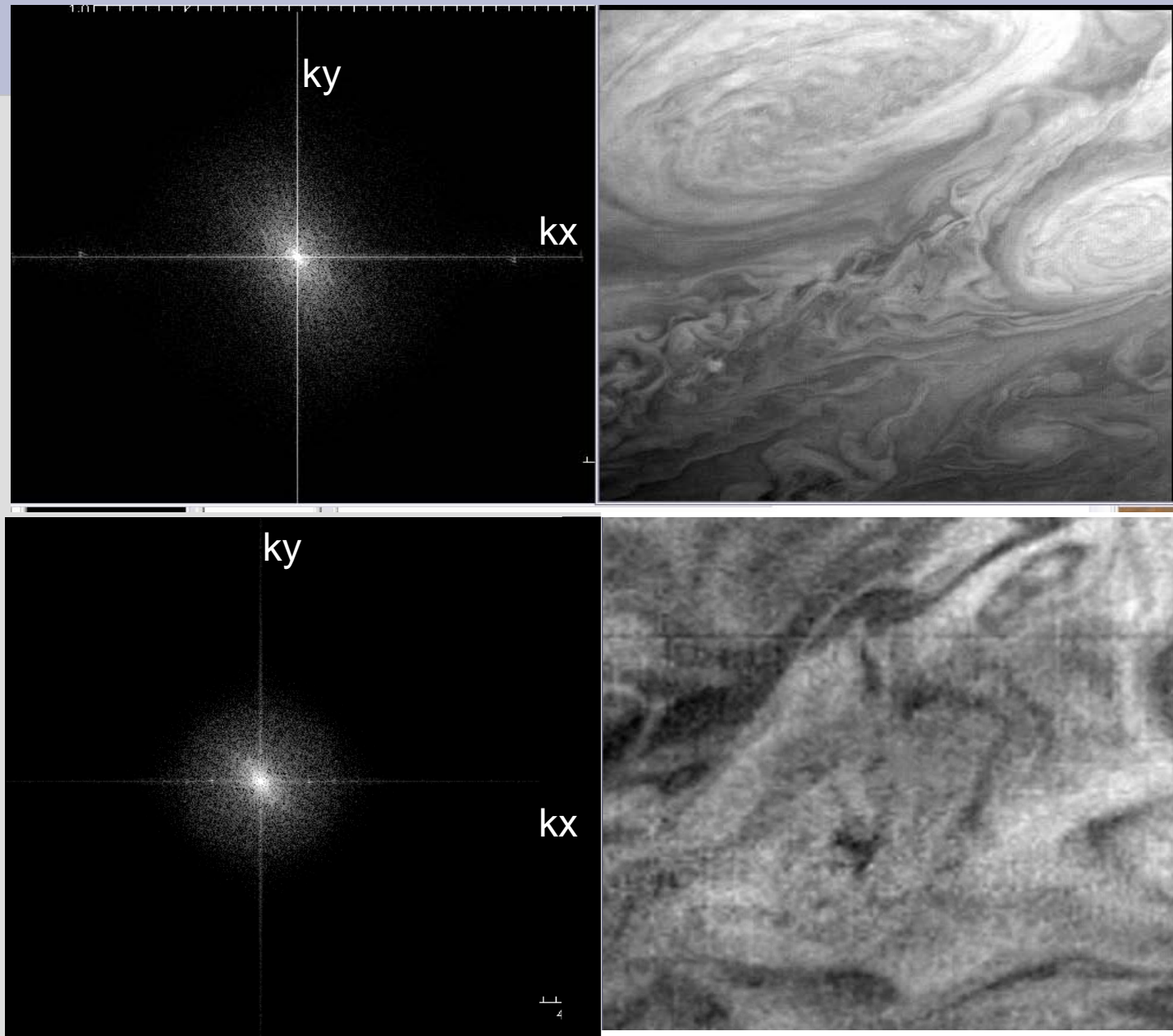
$$\eta / I_0 \sim \text{Re}^{-3/4} \sim 10^9$$

Energy spectrum would have a long inertial subrange!  $k^{(-5/3)}$ ?

Higher resolution images not available.

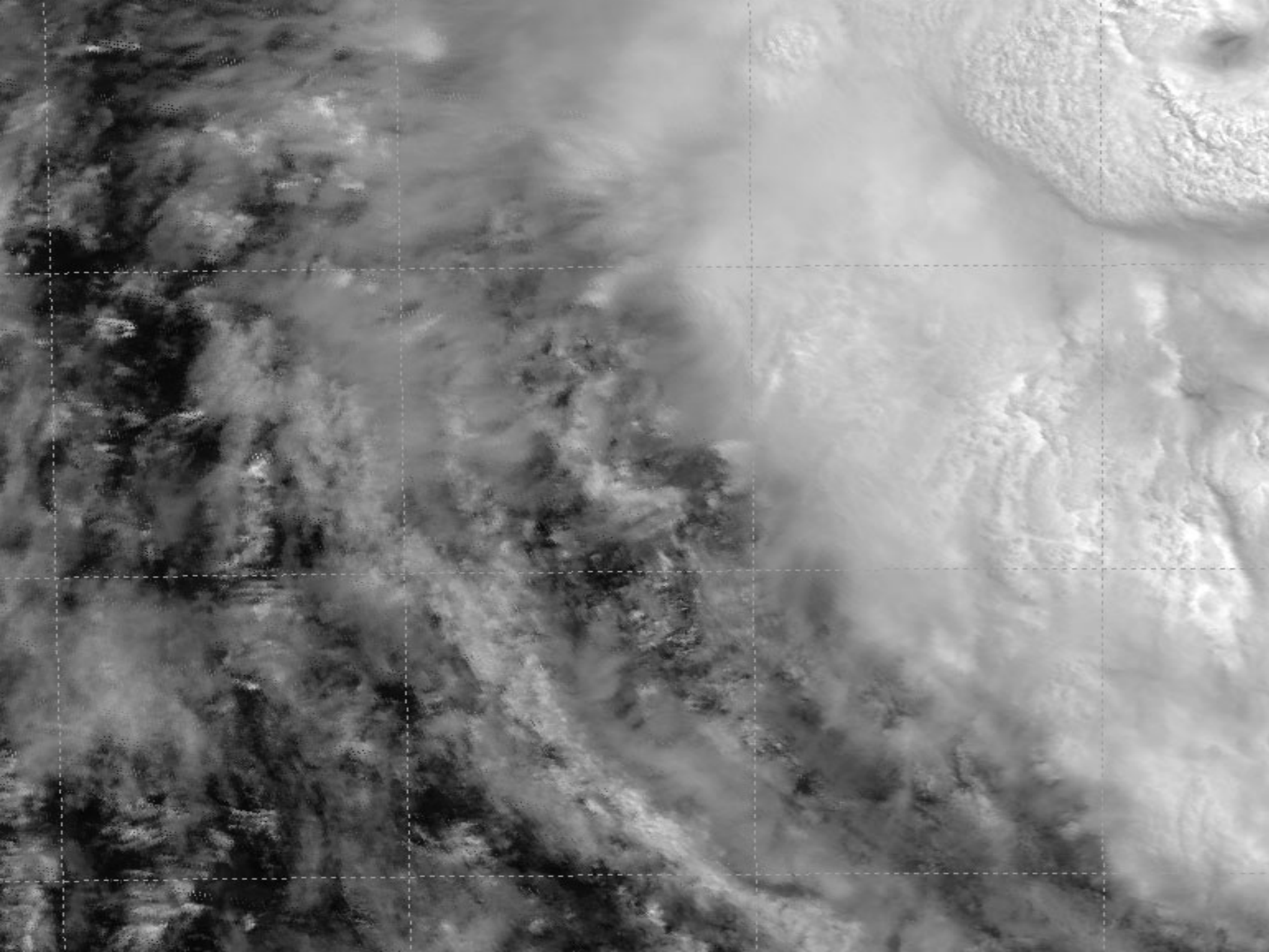


# 2D Fourier Transform of the images for qualitative comparison in k-space



Fourier Transform Method:

- Blue Channel
- Demeaned
- Windowed with cosine taper 15% of image
- DC component shifted to the center
- Axes extend to Nyquist
- Absolute value plotted, with brighter representing more amplitude






# Solar Granulation

David Beaman

MAE 252B

May 13, 2008



Thermal instabilities in the upper portion of the convective zone give rise to an unstable turbulent motion which controls the transport of heat to the surface of the Sun.

## Granules (C)

- Length Scale  $\sim 1000\text{km}$  (size of Texas)
- Time Scale  $\sim 10$  minutes
- Velocity Scale  $\sim 1\text{-}2\text{km}\cdot\text{s}^{-1}$
- $Re > 10^{12}$
- Large scale coherence emerges from small scale turbulent dynamics (inverse cascade)

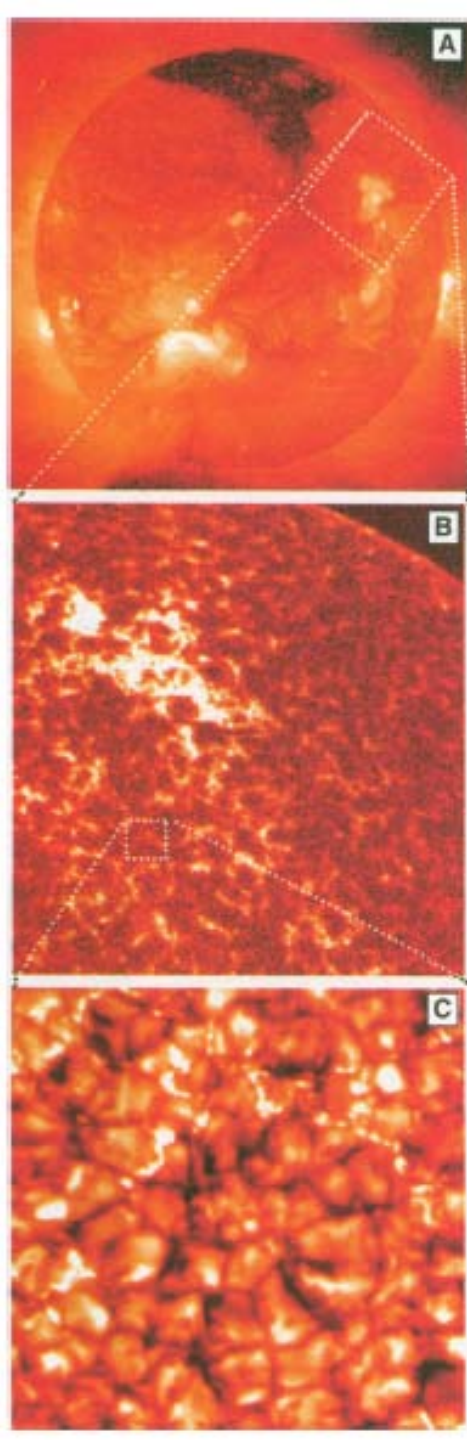
## Supergranules (B)

- $\sim 30,000\text{km}$  diameter
- Last anywhere from a day to a week
- $\sim$  same velocities as granules

(A) Image of Solar atmosphere from the Soft X-Ray Telescope

(B) Strong horizontal flows of supergranulation

(C) Solar granules

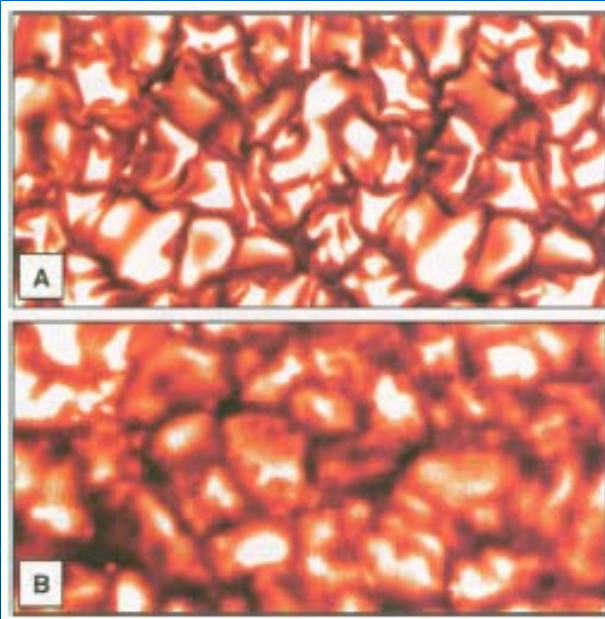


## Solar Numerical Simulations

- Active scales of Solar turbulence:  
 $\sim 10^{-1} - 10^5$  km
- # of grid points (LES),  $N \sim 10^{30}$   
while computers can only handle  
 $N \sim 10^{12}$
- LES resolves only fraction of scales

Complex issues of solving the photosphere's turbulence (not textbook case of isotropic incompressible Kolmogorov turbulence):

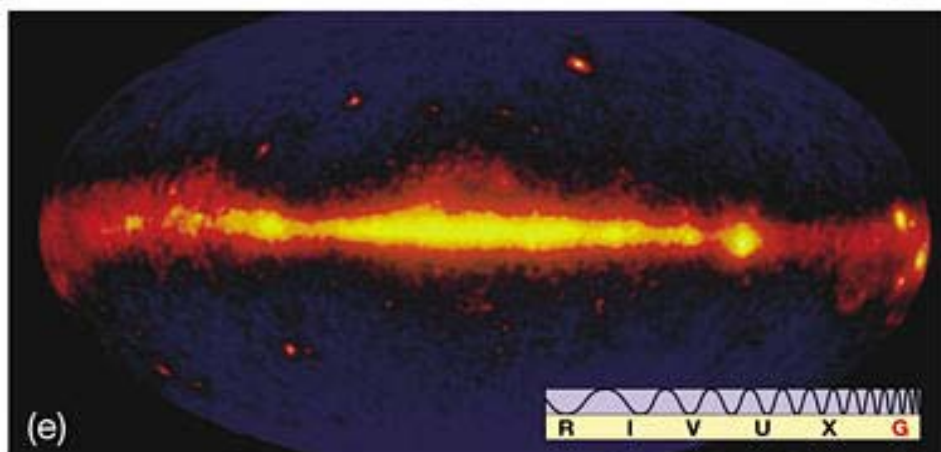
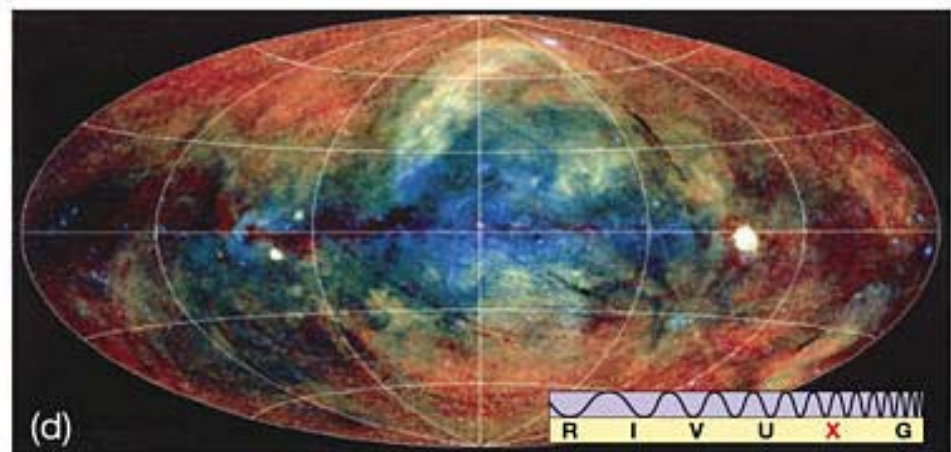
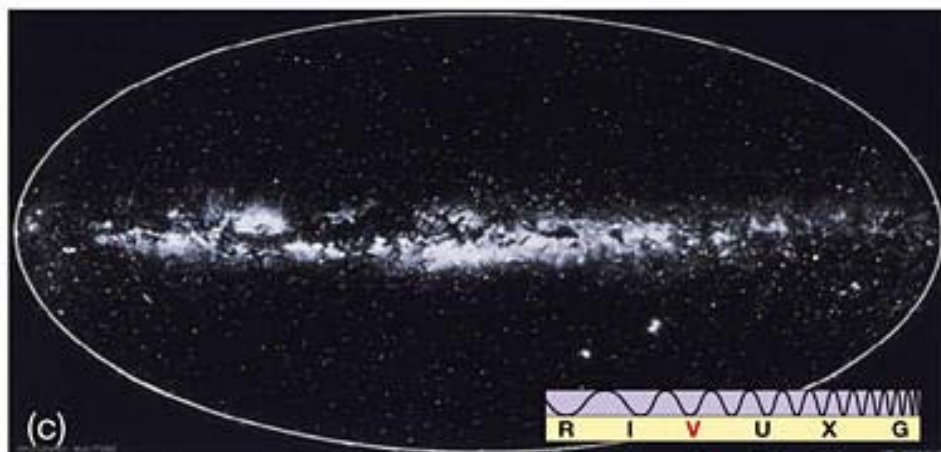
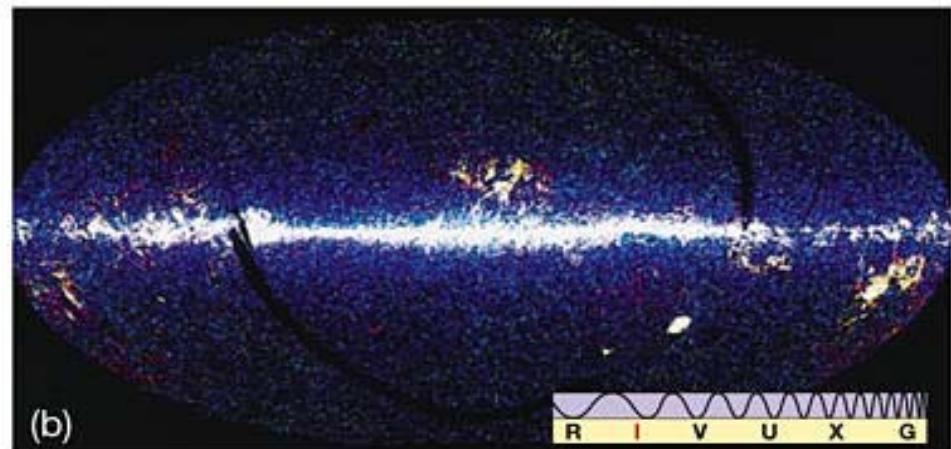
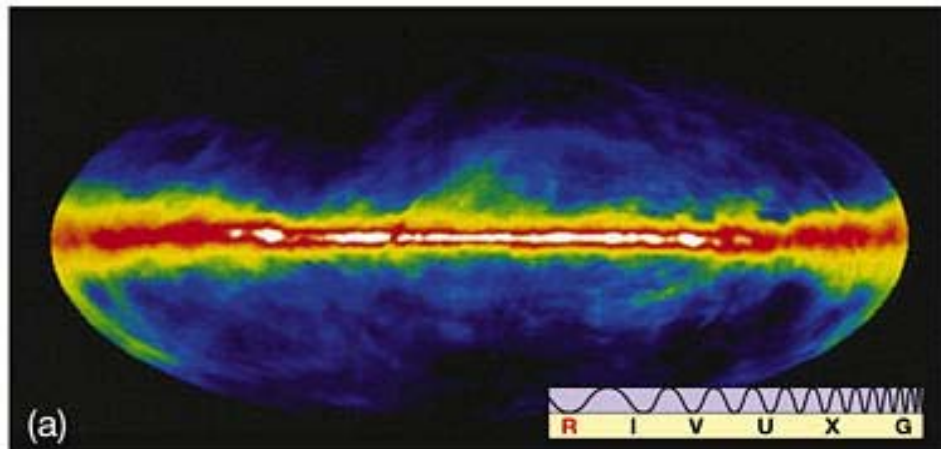
- Inhomogeneity and Anisotropy
- Compressibility effects
- Intermittency (alterations of phases)
- Effects of 3-D MHD



(A) *Simulation* of solar granulation of intensity  
Domain: 6000km (horiz) x 3000km (vert)

(B) Same scale image of observed intensity: La Palma, Sweden.

Note: if the simulated were adjusted to account for atmospheric seeing distortions, the two images would be qualitatively similar.



Images of the Milky Way in different wavelengths

**QUESTIONS?**



A dramatic night scene of a volcano erupting. A massive, bright yellow and orange lava flow is visible at the bottom, with a large plume of dark smoke rising from it. A powerful lightning bolt strikes the sky above the volcano, illuminating the scene. The background is dark, suggesting a night sky.

# Volcanoes

John Douglass

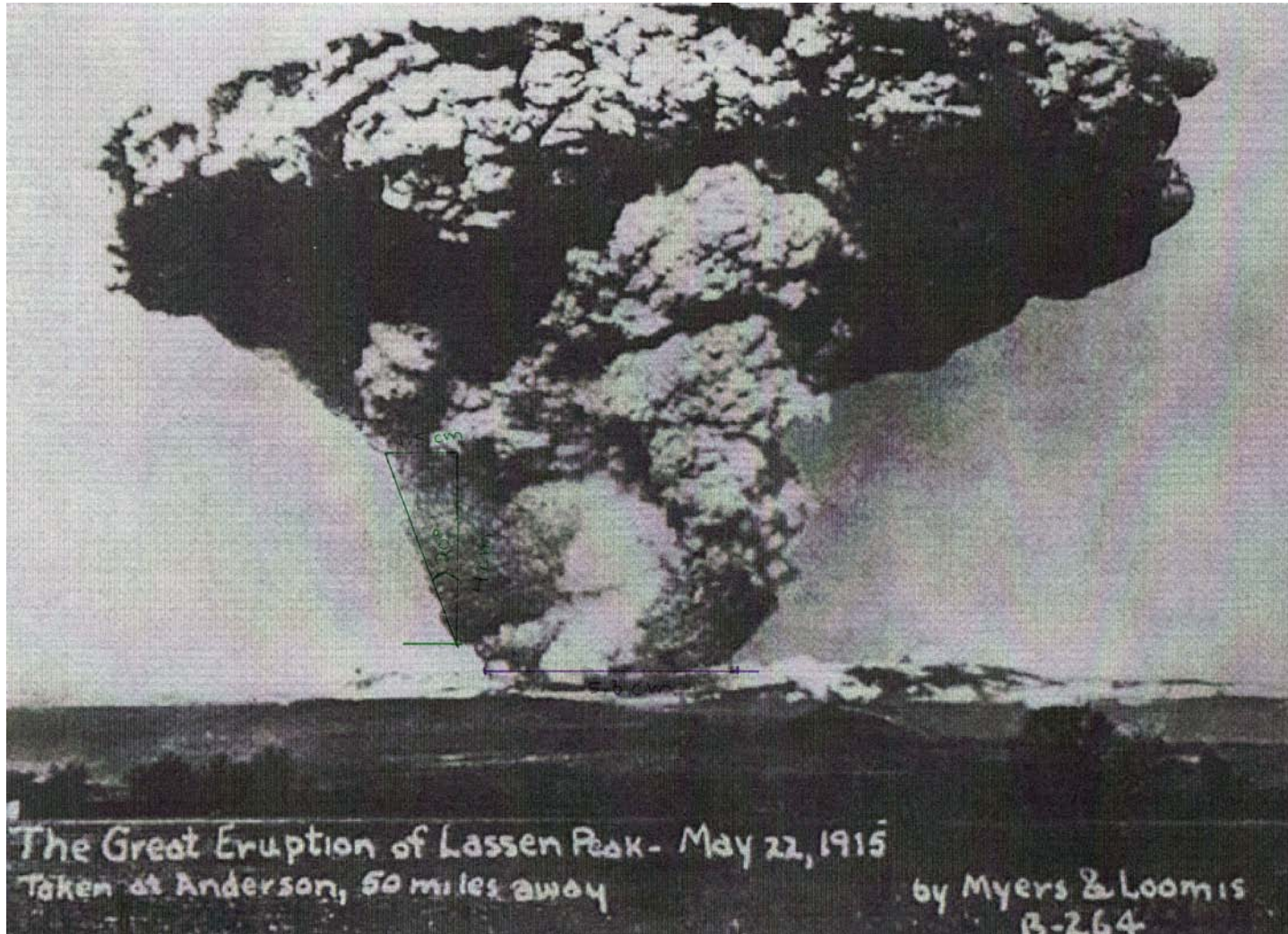
# Saint Helens (1980)



# Lassen Peak (1915)



# Lassen Peak (1915)







- Take or collect interesting images of turbulent flows, and explain them with the knowledge you have learned from this class:
  - September 27, Monday, about 5 min. each